



SYMBOLIC LOGIC

PART I ELEMENTARY

BY
LEWIS CARROLL

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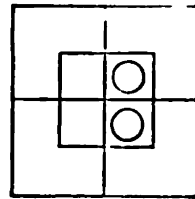
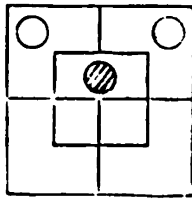
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A Syllogism worked out.

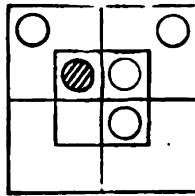
That story of yours, about your once meeting the sea-serpent, always sets me off yawning;

I never yawn, unless when I'm listening to something totally devoid of interest.

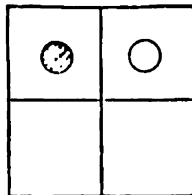
The Premisses, separately.



The Premisses, combined.



The Conclusion.



That story of yours, about your once meeting the sea-serpent, is totally devoid of interest.

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ADVERTISEMENT.

I SHALL be grateful to any Reader who will point out any mistakes or misprints he may happen to notice in this book, or any passage which he thinks is not clearly expressed.

I have a quantity of MS. on hand for Parts II and III, and hope to be able——should life, and health, and opportunity, be granted to me, to publish them in the course of the next few years. Their contents will be as follows:—

PART II. ADVANCED.

Further investigations in the subjects of Part I. Propositions of other forms (such as “Not-all x are y ”). Triliteral and Multiliteral Propositions (such as “All abc are de ”). Hypotheticals. Dilemmas. Paradoxes. &c. &c.

PART III. TRANSCENDENTAL.

Analysis of a Proposition into its Elements. Numerical and Geometrical Problems. And many other *Curiosa Logica*.

Dedicated
to
the Memory
of
Aristotle

INTRODUCTION.

TO LEARNERS.

[N.B. Some remarks, addressed to *Teachers*, will be found in the Appendix, at p. 163.]

THE Learner, who wishes to try the question *fairly*, whether this little book does, or does not, supply the materials for a most interesting mental recreation, is *earnestly* advised to adopt the following Rules:—

(1) Begin at the *beginning*, and do not allow yourself to gratify a mere idle curiosity by dipping into the book, here and there. This would very likely lead to your throwing it aside, with the remark “This is *much* too hard for me!”, and thus losing the chance of adding a very *large* item to your stock of mental delights. This Rule (of not *dipping*) is very *desirable* with *other* kinds of books—such as novels, for instance, where you may easily spoil much of the enjoyment you would otherwise get from the story, by dipping into it further on, so that what the author meant to be a pleasant surprise comes to you as a matter of course. Some people, I know, make a practice of looking into Vol. III first, just to see how the story ends: and perhaps it *is* as well just to know that all ends *happily*—that the much-persecuted lovers *do* marry after all, that he is proved to be quite innocent of the murder, that the wicked cousin is completely foiled in his plot and gets the punishment he deserves, and that the rich uncle in India (*Qu. Why in India? Ans. Because, somehow, uncles never can get rich anywhere else*) dies at exactly the right moment—before taking the trouble to read Vol. I. This, I say, is *just* permissible with a *novel*, where Vol. III has a *meaning*, even for those who have not read the earlier part of the story; but, with a *scientific* book, it is sheer insanity: you will find the latter part *hopelessly* unintelligible, if you read it before reaching it in regular course.

(2) Don't begin any fresh Chapter, or Section, until you are certain that you *thoroughly* understand the whole book *up to that point*, and that you have worked, correctly, most if not all of the examples which have been set. So long as you are conscious that all the land you have passed through is absolutely *conquered*, and that you are leaving no unsolved difficulties *behind* you, which will be sure to turn up again later on, your triumphal progress will be easy and delightful. Otherwise, you will find your state of puzzlement get worse and worse as you proceed, till you give up the whole thing in utter disgust.

(3) When you come to any passage you don't understand, *read it again*: if you *still* don't understand it, *read it again*: if you fail, even after *three* readings, very likely your brain is getting a little tired. In that case, put the book away, and take to other occupations, and next day, when you come to it fresh, you will very likely find that it is *quite* easy.

(4) If possible, find some genial friend, who will read the book along with you, and will talk over the difficulties with you. *Talking* is a wonderful smoother-over of difficulties. When *I* come upon anything—in Logic or in any other hard subject—that entirely puzzles me, I find it a capital plan to talk it over, *aloud*, even when I am all alone. One can explain things so *clearly* to one's self! And then, you know, one is so *patient* with one's self: one *never* gets irritated at one's own stupidity!

If, dear Reader, you will faithfully observe these Rules, and so give my little book a really *fair* trial, I promise you, most confidently, that you will find Logic to be one of the most—if not *the* most—fascinating of mental recreations! In this First Part, I have carefully avoided all difficulties which seemed to me to be beyond the grasp of an intelligent child of (say) twelve or fourteen years of age. I have myself taught most of its contents, *vivâ voce*, to *many* children, and have found them take a real intelligent interest in the subject. For those, who succeed in mastering Part I, and who begin, like Oliver, “asking for more,” I hope to provide, in Part II, some *tolerably* hard nuts to crack—nuts that will require all the *nut-crackers* they happen to possess!

Mental recreation is a thing that we all of us need for our mental health; and you may get much healthy enjoyment, no doubt, from Games, such as Back-gammon, Chess, and the new Game "Halma". But, after all, when you have made yourself a first-rate player at any one of these Games, you have nothing real to *show* for it, as a *result*! You enjoyed the Game, and the victory, no doubt, *at the time*: but you have no *result* that you can treasure up and get real *good* out of. And, all the while, you have been leaving unexplored a perfect *mine* of wealth. Once master the machinery of Symbolic Logic, and you have a mental occupation always at hand, of absorbing interest, and one that will be of real *use* to you in *any* subject you may take up. It will give you clearness of thought—the ability to *see your way* through a puzzle—the habit of arranging your ideas in an orderly and get-at-able form—and, more valuable than all, the power to detect *fallacies*, and to tear to pieces the flimsy illogical arguments, which you will so continually encounter in books, in newspapers, in speeches, and even in sermons, and which so easily delude those who have never taken the trouble to master this fascinating Art. *Try it.* That is all I ask of you!

L. C.

29, BEDFORD STREET,
COVENT GARDEN.

January, 1896.

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




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BOOK I.

THINGS AND THEIR ATTRIBUTES.

CHAPTER I.

INTRODUCTORY.

THE Universe contains '**Things.**'

[For example, "I," "London," "roses," "redness," "old English books," "the letter which I received yesterday."]

Things have '**Attributes.**'

[For example, "large," "red," "old," "which I received yesterday."]

One Thing may have many Attributes ; and one Attribute may belong to many Things.

[Thus, the Thing "a rose" may have the Attributes "red," "scented," "full-blown," &c. ; and the Attribute "red" may belong to the Things "a rose," "a brick," "a ribbon," &c.]

CHAPTER II.

CLASSIFICATION.

'CLASSIFICATION,' or the formation of Classes, is a Mental Process, in which we imagine that we have picked out, from a certain Set of Things, all that possess a certain Attribute (or Set of Attributes), and have put them together in a group by themselves.

Such a group is called a '**Class**'; and the Attribute (or Set of Attributes) is said to be '**peculiar**' to it.

As this Process is entirely *Mental*, we may perform it without knowing whether there *are*, or *are not*, any existing Things which are Members of it. If there *are*, the Class is said to be '**Real**'; if *not*, it is said to be '**Imaginary**.'

[For example, we might think of the whole Universe, and of the Attribute "valuable," and thus form the Real Class "valuable Things"; i.e. (in one word) "Treasures."

Again, we might think of the whole Universe, and of the Set of Attributes "material, artificial, consisting of houses and streets," and thus form the Real Class "material artificial Things, consisting of houses and streets"; i.e. (in one word) "Towns."

Again, we might think of the Set of Things "Towns," and of the Set of Attributes "English, having four million inhabitants," and thus form the Real Class "English Towns having four million inhabitants."

Again, we might think of the Set of Things "Towns," and of the Set of Attributes "English, having *ten* million inhabitants," and thus form the Imaginary Class "English Town having ten million inhabitants."]

A Class, containing only *one* Member, is called an **'Individual.'**

[For example, the Class "English Towns having four million inhabitants," which Class contains only *one* Member, viz. "London."]

Hence, any single Thing, which we can name so as to distinguish it from all other Things, may be regarded as a one-member Class.

[Thus "London" may be regarded as the one-Member Class, picked out from the Set of Things "Towns," which has, as its peculiar Attribute, either "having four million inhabitants" or "which we refer to when we use the Name 'London'" or any other Attribute (or Set of Attributes) which will serve to distinguish it from all other Towns.]

A Class, containing two or more Members, is sometimes regarded as *one single Thing*. When so regarded, it may possess an Attribute (or Set of Attributes) which is *not* possessed by any Member of it taken separately.

[Thus, the Class "The Soldiers of the Tenth Regiment," when regarded as *one single Thing*, may possess the Attribute "formed in square," which is *not* possessed by any Member of it taken separately.]

CHAPTER III.

DIVISION.

§ 1.

Introductory.

‘DIVISION’ is a Mental Process, in which we think of a certain Set of Things, and imagine that we have divided it into two or more Classes.

[Thus, we might think of the Set of Things “books,” and imagine that we had divided it into the two Classes “bound books” and “unbound books,” or into the three Classes, “books priced at less than a shilling,” “shilling-books,” “books priced at more than a shilling,” or into the twenty-six Classes, “books whose names begin with *A*,” “books whose names begin with *B*,” &c.]

§ 2.

Dichotomy.

‘DICHOTOMY’ is Division into *two* Classes.

If a certain Set of Things be divided into two Classes, one of which contains all the Things possessing a certain Attribute (or Set of Attributes), and the other all the Things *not* possessing it (that is, all the rest of the Set), the Process is called ‘**Dichotomy by Contradiction.**’

[For example, we may divide “books” into the two Classes, “books that possess the Attribute ‘old’,” and “books that do *not* possess it.”]

In such a case, it is convenient to regard *each* of the two Classes as having its own *peculiar* Attribute—the first having the Attribute used in the Process of Division, and the second having the same, with the word “not” prefixed.

[Thus we may divide “books” into the two Classes, whose peculiar Attributes are “old” and “not-old.”]

In performing this Process, we may sometimes find that the Attribute we have chosen is used so loosely, in ordinary conversation, that it is not easy to decide *which* of the Things belong to the one Class and *which* to the other. In such a case, it would be necessary to lay down some arbitrary *rule*, as to *where* the one Class should end and the other begin.

[Thus, in dividing "books" into "old" and "not-old," we may say "Let all books printed before A.D. 1801, be regarded as "old," and all others as "not-old.""]

Henceforwards let it be understood that, if a Set of Things be divided into two Classes, whose peculiar Attributes have contrary meanings, each is to be regarded as equivalent to the other with the word "not" prefixed.

[Thus, if "books" be divided into "old" and "new," the word "old" is to be regarded as equivalent to "not-new," and the word "new" as equivalent to "not-old."]

After dividing a Set of Things, by the Process of *Dichotomy by Contradiction*, into two Classes, we may sub-divide each of these into two Classes; and this Process may be repeated over and over again, the number of Classes being doubled at each repetition.

[For example, we may divide "books" into "old" and "not-old" (i.e. "new"): we may then sub-divide each of these into "English" and "not-English" (i.e. "foreign"), thus getting *four* Classes, viz.

- (1) old English;
- (2) old foreign;
- (3) new English;
- (4) new foreign.

If we had begun by dividing into "English" and "foreign," and had then sub-divided into "old" and "new," the four Classes would have been

- (1) English old;
- (2) English new;
- (3) foreign old;
- (4) foreign new.

The Reader will easily see that these are the very same four Classes which we had before.]

CHAPTER IV.

NAMES.

THE word "Thing", which conveys the idea of a Thing, *without* any idea of an Attribute, represents *any* Thing in the Universe. Any other word (or phrase), which conveys the idea of a Thing, *with* the idea of an Attribute (or Set of Attributes), represents *any* Thing possessing that Attribute (or Set of Attributes).

Such a word (or phrase) is called a '**Name**' of the Thing which it represents.

[For example, the words "Thing," "Treasure," "Town," and the phrases "valuable Thing," "material artificial Thing consisting of houses and streets," "English Town having four million inhabitants," "English Town having ten million inhabitants," "old English Book."]

Every Name is either a Substantive, or a phrase consisting of a Substantive and one or more Adjectives (or phrases used as Adjectives).

Every Name, except "Thing," may usually be expressed in any one of the following three different forms:—

- (a) The Substantive "Thing," and one or more Adjectives (or phrases used as Adjectives) conveying the ideas of the Attributes ;
- (b) A Substantive, conveying the idea of a Thing with the ideas of *some* of the Attributes, and one or more Adjectives (or phrases used as Adjectives) conveying the ideas of the *other* Attributes ;

(c) A Substantive conveying the idea of a Thing with the ideas of *all* the Attributes.

[Thus, the phrase "material living Thing, belonging to the Animal Kingdom, having two hands and two feet" is a Name expressed in Form (a).

If we choose to roll up together the Substantive "Thing" and the Adjectives "material, living, belonging to the Animal Kingdom," so as to make the new Substantive "Animal," we get the phrase "Animal having two hands and two feet," which is a Name (representing the same Thing as before) expressed in Form (b).

And if we choose to roll up the whole phrase into one word, so as to make the new Substantive "Man," we get a Name (still representing the very same Thing) expressed in Form (c).]

A Name, whose Substantive is in the *plural* number, may be used to represent either

- (1) Members of a Class, *regarded as separate Things* ;
- or (2) a whole Class, *regarded as one single Thing*.

[Thus, when I say "Some Soldiers of the Tenth Regiment are tall," or "The Soldiers of the Tenth Regiment are brave," I am using the Name "Soldiers of the Tenth Regiment" in the *first* sense ; and it is just the same as if I were to point to each of them *separately*, and to say "*This* Soldier of the Tenth Regiment is tall," "*That* Soldier of the Tenth Regiment is tall," and so on.

But, when I say "The Soldiers of the Tenth Regiment are formed in square," I am using the phrase in the *second* sense ; and it is just the same as if I were to say "The *Tenth Regiment* is formed in square."]

CHAPTER V.

DEFINITIONS.

WHEN two Sets of Things are such that the second may be regarded as a Class picked out from the first, the first is called a '**Genus**' with regard to the second, the second is called a '**Species**' of the first, and its peculiar Attribute (or Set of Attributes) is called its '**Differentia**'.

[Thus, in the examples given in Chapter II., we may regard "Things" as a *Genus*, "Treasures" as a *Species* of "Things," and "valuable" as its *Differentia*.

Again, we may regard "Things" as a *Genus*, "Towns" as a *Species* of Things, and "material, artificial, consisting of houses and streets" as its *Differentia*.

Again, we may regard "Towns" as a *Genus*, "English Towns having four million inhabitants" as a *Species* of Towns, and "English, having four million inhabitants" as its *Differentia*.]

It is evident that any Member of a *Species* is *also* a Member of the *Genus* out of which that Species has been picked, and that it possesses the *Differentia* of that Species. Hence any Member of a Species may be represented by a Name consisting of two parts, one being a Name representing any Member of the *Genus*, and the other being the *Differentia* of that Species. Such a Name is called a '**Definition**' of any Member of that Species, and to give it such a Name is to '**define**' it.

[Thus, we may define a "Treasure" as a "valuable Thing." In this case we regard "Things" as the *Genus*, and "valuable" as the *Differentia*.]

The following Examples, of this Process, may be taken as models for working others.

[Note that, in each Definition, the Substantive, representing a Member (or Members) of the *Genus*, is printed in Capitals.]

1. Define "a Treasure."

Ans. "a valuable THING."

2. Define "Treasures."

Ans. "valuable THINGS."

3. Define "a Town."

Ans. "a material artificial THING, consisting of houses and streets."

4. Define "Men."

Ans. "material, living THINGS, belonging to the Animal Kingdom, having two hands and two feet."

or else

"ANIMALS having two hands and two feet."

[The Reader can set himself any number of Examples of this Process, by simply choosing the Name of any common Thing (such as "house," "tree," "knife"), making a Definition for it, and then testing his answer by referring to any English Dictionary.]

BOOK II.

PROPOSITIONS.

CHAPTER I.

INTRODUCTORY.

THE word 'Proposition,' as used in ordinary conversation, may be applied to *any* word, or phrase, which conveys any information whatever.

[Thus the words "yes" and "no" are Propositions in the ordinary sense of the word ; and so are the phrases "you owe me five farthings," and "I don't !"]

Such words as "oh !" or 'never !', and such phrases as "fetch me that book !" "which book do you mean ?" do not seem, at first sight, to convey any *information* : but they can easily be turned into equivalent forms which do so, viz. "I am surprised," "I will never consent to it," "I order you to fetch me that book," "I want to know which book you mean."]

But a '**Proposition**,' as used in Logic, has a peculiar form, which may be called '**Normal**' ; and if any Proposition, which we wish to use in an argument, is not in normal form, we must reduce it to such a form, before we can use it.

Propositions are of two kinds, 'Propositions of Existence' and 'Propositions of Relation.'

These shall be discussed separately.

CHAPTER II.

PROPOSITIONS OF EXISTENCE

§ 1.

Introductory.

NOTE that the word “some ” is to be regarded, henceforward, as meaning “one or more.”

A ‘ **Proposition of Existence** ’ affirms, as to a certain Class, which is called its ‘ **Subject**,’ either that there *are*, or else that there *are not*, existing Things which are Members of it.

That is, it affirms either that its Subject is *Real*, or else that it is *Imaginary*.

[Thus, if the Subject were “honest men,” the Proposition would affirm either that there *are*, or else that there *are not*, existing honest men.

In the first case, it might be expressed in the form
“Some honest men exist”.

In the second case, it might be expressed in the form
“No honest men exist”.]

§ 2.

Normal form of a Proposition of Existence.

A Proposition of Existence, in normal form, consists of three parts, viz.

(1) The word "some" or "no." (This word, which tells us *how many* existing Things are Members of the Subject, is called the '**Sign of Quantity.**')
 [Note that it does not tell us the *exact* number : in fact, it only deals with *two* numbers, which are (in ascending order) "0" and "1 or more."]

(2) The Name of the Subject.

(3) The Verb denoting existence.

[The Propositions, given in the preceding Section, may be arranged thus :—

"Some honest men exist"

"Some" *Sign of Quantity.*

"honest men" *Subject.*

"exist" *Verb.*

or thus :—

"Some | honest men | exist."

"No honest men exist."

"No" *Sign of Quantity.*

"honest men" *Subject.*

"exist" *Verb.*

or thus :—

"No | honest men | exist."]

§ 3.

Reduction of a Proposition of Existence to Normal form.

THE Rules, to be observed, are :—

(1) Ascertain the Name of the *Subject*, by picking out the Substantive representing the Set of Things referred to, and *arranging* the rest of the sentence as a Set of Attributes.

(2) Ascertain the *Sign of Quantity.*

(3) Arrange in the following order :—

Sign of Quantity.

Subject.

The verb “exist.”

[For example, “Some apples are not ripe.” Here we pick out “apples” as the Substantive required, and “not-ripe” as its Attribute ; and we arrange thus :—

“Some” *Sign of Quantity.*

“not-ripe apples” *Subject.*

“exist” *Verb.*

or thus :—

“Some | not-ripe apples | exist.”.

Again, “No lambs are accustomed to smoke cigars.” Here we pick out “lambs” as the Substantive, and “accustomed to smoke cigars” as its Attribute ; and we arrange thus :—

“No” *Sign of Quantity.*

“lambs accustomed to

smoke cigars *Subject.*

“exist” *Verb.*

or thus :—

“No | lambs accustomed to smoke cigars | exist.

CHAPTER III.
PROPOSITIONS OF RELATION.

§ 1.

Introductory.

A ‘**Proposition of Relation**’ affirms, as to certain two Classes, which are Species of the *same Genus*, and which are called its ‘**Subject**’ and ‘**Predicate**,’ either

- (1) that *some* Members of its Subject are Members of its Predicate ;
- or (2) that *no* Members of its Subject are Members of its Predicate ;
- or (3) that *all* Members of its Subject are Members of its Predicate.

The Genus referred to is called the ‘**Universe of Discourse**,’ or (more briefly) ‘**Univ.**’

[Thus, if Univ. were “books,” and if the Subject were “French books” and the Predicate “bound books,” the Proposition would affirm either

- (1) “Some French books are bound (i.e. bound books)” ;
- or (2) “No French books are bound” ;
- or (3) “All French books are bound.”]

The Subject and the Predicate of a Proposition of Relation are called its ‘**Terms**.’

§ 2.

Various kinds of Propositions of Relation, &c.

A Proposition of Relation, that begins with "Some," is said to be '**Particular.**' It is also called a '**Proposition in I.**'

[Note, that it is called 'Particular,' because it refers to a *part* only of its Subject, which it affirms to be included in its Predicate.]

A Proposition of Relation, that begins with "No," is said to be '**Universal Negative.**' It is also called a '**Proposition in E.**'

[Note, that it is called 'Universal Negative,' because it *denies*, as to *every* Member of its Subject, that it is a Member of its Predicate.]

A Proposition of Relation, that begins with "All," is said to be '**Universal Affirmative.**' It is also called a '**Proposition in A.**'

[Note, that it is called 'Universal Affirmative,' because it *affirms*, as to *every* Member of its Subject, that it is a Member of its Predicate.]

It sometimes happens that, in one or both of the Terms of a Proposition of Relation, the Name consists of Adjectives only, the Substantive being understood. In order to express such a Proposition *fully*, we must supply the Name of whatever Set of Things seems to have been regarded as Univ.

[Thus, the Propositions "Some apples are not ripe," "No lambs are accustomed to smoke cigars," "Brave soldiers deserve honour," "The wise are listened to," "None but monarchs are crowned," may be expressed fully as "Some apples are not-ripe fruit," "No lambs are animals accustomed to smoke cigars," "Brave soldiers are men worthy of honour," "Wise persons are persons who are listened to," "No persons, who are not monarchs, are crowned persons."

But we may not express "Brave soldiers deserve honour" as "Brave soldiers are *Englishmen* deserving of honour," since the idea of the Attribute "English" is *not* conveyed by the phrase "brave soldiers," so that we cannot regard "Englishmen" as Univ.]

§ 3.

Normal form of a Proposition of Relation.

A Proposition of Relation, in normal form, consists of four parts, viz.—

- (1) The word “some,” or “no,” or “all.” (This word, which tells us *how many* Members of the Subject are also Members of the Predicate, is called the ‘**Sign of Quantity.**’)

[Note that it does not tell us the *exact* number: in fact, it only deals with *three* numbers, which are (in ascending order) “0”, “1 or more”, and “the total number of Members of the Subject”.]

- (2) Name of Subject.
- (3) The verb “are.” (This is called the ‘**Copula.**’)
- (4) Name of Predicate.

[The Propositions, given in § 2, may be expressed, in normal form, as follows :—

- (1) “Some apples are not ripe” ;

“Some”	<i>Sign of Quantity.</i>
“apples”	<i>Subject.</i>
“are”	<i>Copula.</i>
“not-ripe fruit”	<i>Predicate.</i>

or thus :—

- “Some | apples | are | not-ripe fruit.”
- (2) “No lambs are accustomed to smoke cigars.”
 “No | lambs | are | animals accustomed to smoke cigars.”
- (3) “Brave soldiers deserve honour.”
 “All | brave soldiers | are | men deserving of honour.”
- (4) “The wise are listened to.”
 “All | wise persons | are | persons who are listened to.”
- (5) “None but monarchs are crowned.”
 “No | persons who are not monarchs | are | crowned persons.”]

A Proposition, whose Subject is an *Individual*, is to be regarded as *Universal*.

[Thus, in the Proposition "John is not well," the Subject the Class "Johns," i.e. the Class of men who are here referred to by the name "John": and this Proposition would be, normal form.

"All" *Sign of Quantity*

"Johns" *Subject.*

"are" *Copula.*

"men who are not well" *Predicate.*

or else

“All | Johns | are | men who are not well.”

§ 4.

Reduction of a Proposition of Relation to Normal form.

THE Rules, for doing this, are as follows:—

(1) If either Term consists of Adjectives only, ascertain what is the *Univ.* intended.

(2) Ascertain the *Sign of Quantity*.

(3) Ascertain what is the *Subject*.

(4) If the verb, governed by the Subject, is *not* the verb “is” (or “are”), substitute for it a phrase consisting of the verb and some Attribute (or Set of Attributes) which will be a part of the Predicate.

(5) Ascertain what is the *Predicate*.

(6) Arrange in the following order :—

Sign of Quantity,

Subject,

Copula,

Predicate.

[Let us work a few Examples, to illustrate these Rules.

(1)

“Some farmers always grumble at the weather, whatever may happen to be.”

(1) Let Univ. be "persons."

(2) The Sign of Quantity is "some."

- (3) The Subject is "farmers."
- (4) The verb is "grumble," for which we substitute the phrase "are persons who grumble."
- (5) The Predicate is "persons who always grumble &c."
- (6) The Proposition now becomes
 "Some | farmers | are | persons who always
 grumble at the weather, whatever it may happen
 to be."

(2)

"None of my speculations have brought me as much as 5 per cent."

- (1) Let Univ. be "transactions."
- (2) The Sign of Quantity is "no."
- (3) The Subject is "my speculations."
- (4) The verb is "have brought," for which we substitute the phrase "are transactions that have brought."
- (5) The Predicate is "transactions that have brought &c."
- (6) The Proposition now becomes
 "None of | my speculations | are | transactions
 that have brought me as much as 5 per cent."

(3)

"My carriage will meet you at the station and take you to the house."

- (1) Let Univ. be "things."
- (2) The Sign of Quantity is "all."
- (3) The Subject is "my carriage." This, being an 'Individual,' is equivalent to "the Class of Things referred to by the phrase 'my carriage,'" or to "my carriages." (Note that this Class contains only *one* Member.)
- (4) The verbs (for there are *two*) are "will meet" and "will take," for which we may substitute the phrase "are things that will meet &c. and will take &c."
- (5) The Predicate is "things that will meet &c. and will take &c."
- (6) The Proposition now becomes
 "All | my carriages | are | things that will meet
 you at the station and take you to the house."

(4)

"Happy is the man who does not know what 'toothache means!'"

- (1) Let Univ. be "men."
- (2) The Sign of Quantity is "all."
- (3) The Subject is evidently "the man &c." (Note that in this sentence, the *Predicate* comes first.) At first sight, this seems to be an 'Individual'; but, on further consideration, we see that it does *not* imply that there is only *one such man*. Hence it is equivalent to "all men who &c."

(4) The verb is "are."

(5) The Predicate has for its *Attribute* "happy." But no *Substantive* is expressed. Let us supply "men."

(6) The Proposition now becomes

"All | men who do not know what 'toothache' means | are | happy men."

(5)

"None but the brave deserve the fair."

To begin with, we note that the phrase "none but the brave" is equivalent to "no *not-brave*."

(1) Let Univ. be "persons."

(2) The Sign of Quantity is "no."

(3) The Subject has for its *Attribute* "not-brave." But no *Substantive* is expressed. Let us supply "persons."

(4) The verb is "deserve," for which we may substitute the phrase "are deserving of."

(5) The Predicate now has for its *Attribute* the phrase "deserving of the fair." But no *Substantive* is expressed. Let us supply "persons."

(6) The Proposition now becomes

"No | not-brave persons | are | persons deserving of the fair."

(6)

"Katharizon Soap is the only kind I ever use."

You imagine (do you not?) that "Katharizon Soap" is the Subject, and "the only kind I ever use" the Predicate.

This, however, is not the case. The Univ. is, evidently, "kinds of soap"; and it is not my object to give you information about *Katharizon Soap*, but rather about *my own habits*. Hence my *Subject* is the one-Member Class "kinds of soap used by me," and the logical effect of the word "only" is to tell you that *all* Members of this Class are Members of the one-Member Class "Katharizon Soaps." Hence we proceed as follows:—

(1) Let Univ. be "kinds of soap."

(2) The Sign of Quantity is "all."

(3) The Subject has for its *Attribute* "used by me." But no *Substantive* is expressed. Let us supply "kinds of soap."

(4) The Verb is "are."

(5) The Predicate is "Katharizon Soaps."

(6) The Proposition now becomes

"All | kinds of soap used by me | are | Katharizon soaps."]

§ 5.

What is implied, in a Proposition of Relation, as to the existence of its Terms?

NOTE that a Proposition of Relation, beginning with "Some" or "All", is henceforwards to be understood as implying that its *Subject exists*——and therefore, of course, that its *Predicate exists* also.

[Thus, the Proposition "Some farmers are persons who always grumble at the weather, whatever it may happen to be" is to be understood as implying that

(1) The Class "farmers" *exists*.

(2) The Class "persons who always &c." *exists*.

Again, the Proposition "All men, who do not know what 'toothache' means, are happy men" is to be understood as implying that

(1) The Class "men who do not &c" *exists*.

(2) The Class "happy men" *exists*.]

Note also that a Proposition of Relation, beginning with "No", is henceforwards to be understood as not implying anything at all as to the *existence*, or *non-existence*, of its *Subject*, or of its *Predicate*.

[Thus, in each of the four Propositions, "No thieves are respectable persons" (where both Terms happen to *exist*), "No magistrates are men with three heads" (where the Subject happens to be *existent* and the Predicate *non-existent*) "No mermaids are governesses" (where the Subject happens to be *non-existent* and the Predicate *existent*), "No mermaids are creatures with three heads" (where both Terms happen to be *non-existent*), we get all our information, as to the *existence*, or *non-existence*, of the Subject, or of the Predicate, *elsewhere*: the Proposition itself tells us nothing about it.]

[Work Examples § 1, 1—4 (p. 97).]

BOOK III.

THE BILITERAL DIAGRAM.

xy	xy'
$x'y$	$x'y'$

CHAPTER I.

SYMBOLS AND CELLS.

FIRST, let us suppose that the above Diagram is an enclosure assigned to a certain Set of Things.

[The Reader is strongly advised, in reading this Chapter, *not* to refer to the above Diagram, but to draw a large one for himself, *without any letters*, and to have it by him while he reads, and keep his finger on that particular *part* of it, about which he is reading.]

This Set of Things may be either the whole Universe, or else some selected Set of Things, which we may call the ‘Universe of Discourse,’ or, more briefly, ‘Univ.’

[For example, we might say “Let Univ. be books”; and we might imagine the Diagram to be a large table, assigned to all “books.”]

Secondly, let us suppose that we have selected a certain Attribute (or Set of Attributes regarded as a single Attribute), which we may call " x ," and have divided the Univ. into the two Classes whose peculiar Attributes are " x " and " $\text{not-}x$ " (which we may call " x' "), and that we have assigned the *North* Half of the Diagram to the one (which we may call "the Class of x -Things," or "the x -Class"), and the *South* Half to the other (which we may call "the Class of x' -Things," or "the x' -Class").

[For example, we might say "Let x mean 'old,' so that x' will mean 'new'," and we might suppose that we had divided books into the two Classes whose peculiar Attributes are "old" and "new," and had assigned the *North* Half of the table to "*old books*" and the *South* Half to "*new books*."

Thirdly, let us suppose that we have selected another Attribute (or Set of Attributes regarded as a single Attribute), which we may call " y ," and have subdivided the x -Class into the two Classes whose peculiar Attributes are " y " and " y' ," and that we have assigned the *North-West* Cell to the one (which we may call "the xy -Class"), and the *North-East* Cell to the other (which we may call "the xy' -Class").

[For example, we might say "Let y mean 'English,' so that y' will mean 'foreign'," and we might suppose that we had subdivided "*old books*" into the two Classes whose peculiar Attributes are "English" and "foreign," and had assigned the *North-West* Cell to "*old English books*," and the *North-East* Cell to "*old foreign books*."]

Fourthly, let us suppose that we have subdivided the x' -Class in the same manner, and have assigned the *South-West* Cell to the $x'y$ -Class, and the *South-East* Cell to the $x'y'$ -Class.

[For example, we might suppose that we had subdivided "*new books*" into the two Classes "*new English books*" and "*new foreign books*," and had assigned the *South-West* Cell to the one, and the *South-East* Cell to the other.]

It is evident that, if we had begun by dividing for y and y' , and had then subdivided for x and x' , we should have got the *same* four Classes. Hence we see that we have assigned the *West Half* to the y -Class, and the *East Half* to the y' -Class.

[Thus, in the above Example, we should find that we had assigned the *West Half* of the table to “*English books*” and the *East Half* to “*foreign books*.”

We have, in fact, assigned the four Quarters of the table to four different Classes of books, as here shown.

old English books	old foreign books
new English books	new foreign books

The Reader should carefully remember that, in such a phrase as “the x -Things,” the word “Things” means that particular *kind* of Things, to which the whole Diagram has been assigned, that is, the Things selected as Univ.

[Thus, if we say “Let Univ. be ‘books’,” we mean that we have assigned the whole Diagram to “books.” In that case, if we took “ x ” to mean “old,” the phrase “the x -Things” would mean “the old books.”]

The Reader should not go on to the next Chapter until he is *quite familiar* with the *blank* Diagram I have advised him to draw.

He ought to be able to name, *instantly*, the *Attribute* (or Set of *Attributes*) assigned to any Compartment named in the right-hand column of the following Table.

Also he ought to be able to name, *instantly*, the *Compartment* assigned to any Attribute (or Pair of Attributes) named in the left-hand column.

To make sure of this, he had better put the book into the hands of some genial friend, while he himself has nothing but the *blank Diagram*, and get that genial friend to question him

TABLE I.

<i>Attributes of Classes.</i>	<i>Compartments, or Cells, assigned to them.</i>
x	North Half.
x'	South „
y	West „
y'	East „
xy	North-West Cell.
xy'	„ East „
$x'y$	South-West „
$x'y'$	„ East „

on this Table, *dodging* about as much as possible. The Questions and Answers should be something like this:—

Q. “Attribute for West Half?”

A. “ y .”

Q. “Compartment for xy' ?”

A. “North-East Cell.”

Q. “Attributes for South-West Cell?”

A. “ $x'y$.”

&c., &c.

After a little practice, he will find himself able to do without the blank Diagram, and will be able to see it *mentally* (“in my mind’s eye, Horatio!”) while answering the questions of his genial friend. When *this* result has been reached, he may safely go on to the next Chapter.

CHAPTER II.

COUNTERS.

LET us agree that a *Red* Counter, placed within a Cell, shall mean "This Cell is *occupied*" (i.e. "There is at least *one* Thing in it").

Let us also agree that a *Red* Counter, placed on the partition between two Cells, shall mean "The Compartment, made up of these two Cells, is *occupied*; but it is not known *whereabouts*, in it, its occupants are." Hence it may be understood to mean "At least *one* of these two Cells is occupied: possibly *both* are."

Our ingenious American cousins have invented a phrase to describe the condition of a man who has not yet made up his mind *which* of two political parties he will join: such a man is said to be "**sitting on the fence.**" This phrase exactly describes the condition of the Red Counter.

Let us also agree that a *Grey* Counter, placed within a Cell, shall mean "This Cell is *empty*" (i.e. "There is *nothing* in it").

[The Reader had better provide himself with 4 Red Counters and 5 Grey ones.]

CHAPTER III.

REPRESENTATION OF PROPOSITIONS.

§ 1.

Introductory.

HENCEFORWARDS, in stating such Propositions as “Some x -Things exist” or “No x -Things are y ”, I shall omit the word “Things”, which the Reader can supply for himself, and shall write them as “Some x exist” or “No x are y ”.

A Proposition, containing only *one* of the Letters used as Symbols for Attributes, is said to be ‘**Unilateral**’.

[For example, “Some x exist”, “No y exist”, &c.]

A Proposition, containing *two* Letters, is said to be ‘**Bilateral**’.

[For example, “Some xy exist”, “No x are y ”, &c.]

A Proposition is said to be ‘**in terms of**’ the Letters it contains, whether with or without accents.

[Thus, “Some xy exist”, “No x are y ”, &c, are said to be *in terms of* x and y .]

The Propositions, which we shall have to represent, are of two kinds, viz.

(1) Propositions of *Existence* ;

(2) Propositions of *Relation*.

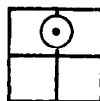
These shall be discussed separately.

§ 2.

Representation of Propositions of Existence.

Let us take, first, the Proposition “Some x exist”.

This tells us that there is at least *one* Thing in the North Half; that is, that the North Half is *occupied*. And this we can evidently represent by placing a *Red* Counter (here represented by a *dotted* circle) on the partition which divides the North Half.



[In the “books” example, this Proposition would be “Some old books exist”.]

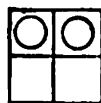
Similarly we may represent the three similar Propositions “Some x' exist”, “Some y exist”, and “Some y' exist”.

[The Reader should make out all these for himself.

In the “books” example, these Propositions would be “Some new books exist”, &c.]

Let us take, next, the Proposition “No x exist”.

This tells us that there is *nothing* in the North Half; that is, that the North Half is *empty*; that is, that the North-West Cell and the North-East Cell are both of them *empty*. And this we can represent by placing *two* *Grey* Counters in the North Half, one in each Cell.



[The Reader may perhaps think that it would be enough to place a *single* *Grey* Counter on the partition dividing the North Half, and that, just as a single *Red* Counter, so placed, would mean “This Half is *occupied*”, so a single *Grey* Counter would mean “This Half is *empty*”.

This, however, would be a mistake. We have seen that a single *Red* Counter, so placed, would mean “At least *one* of these two Cells is occupied: possibly *both* are.” Hence a single *Grey* Counter would merely mean “At least *one* of these two Cells is empty: possibly *both* are”. But “possibly” is not enough: what we have to represent is, that both Cells are *certainly* empty: and this can only be done by placing a *Grey* Counter in *each* of them.

In the “books” example, this Proposition would be “No old books exist”.]

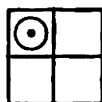
Similarly we may represent the three similar Propositions “No x' exist”, “No y exist”, and “No y' exist”.

[The Reader should make out all these for himself.

In the “books” example, these three Propositions would be “No new books exist”, &c.]

Let us take, next, the Proposition “Some xy exist”.

This tells us that there is at least *one* Thing in the North-West Cell; that is, that the North-West Cell is *occupied*. And this we can represent by placing a *Red* Counter in it.



[In the “books” example, this Proposition would be “Some old English books exist”.]

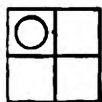
Similarly we may represent the three similar Propositions “Some xy' exist”, “Some $x'y$ exist”, and “Some $x'y'$ exist”.

[The Reader should make out all these for himself.

In the “books” example, these three Propositions would be “Some old foreign books exist”, &c.]

Let us take, next, the Proposition “No xy exist”.

This tells us that there is *nothing* in the North-West Cell; that is, that the North-West Cell is *empty*. And this we can represent by placing a *Grey* Counter in it.



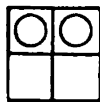
[In the “books” example, this Proposition would be “No old English books exist”.]

Similarly we may represent the three similar Propositions “No xy' exist”, “No $x'y$ exist”, and “No $x'y'$ exist”.

[The Reader should make out all these for himself.

In the “books” example, these three Propositions would be “No old foreign books exist”, &c.]

We have seen that the Proposition “No x exist” may be represented by placing *two Grey Counters* in the North Half, one in each Cell.



We have also seen that these two *Grey Counters*, taken *separately*, represent the two Propositions “No xy exist” and “No xy' exist”.

Hence we see that the Proposition “No x exist” is a *Double Proposition*, and is ‘**equivalent**’ to (*i.e.*, gives the same information as) the *two* Propositions “No xy exist” and “No xy' exist”.

[In the “books” example, this Proposition would be “No old books exist”.

Hence this is a *Double Proposition*, and is equivalent to the *two* Propositions “No old *English* books exist” and “No old *foreign* books exist”.]

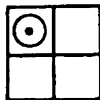
§ 3.

Representation of Propositions of Relation.

Let us take, first, the Proposition “Some x are y ”.

This tells us that at least *one* Thing, in the *North Half*, is also in the *West Half*. Hence it must be in the space *common* to them, that is, in the *North-West Cell*.

Hence the North-West Cell is *occupied*. And this we can represent by placing a *Red Counter* in it.



[Note that the *Subject* of the Proposition settles which *Half* we are to use; and that the *Predicate* settles in which *portion* of it we are to place the Red Counter.

In the “books” example, this Proposition would be “Some old books are English”.]

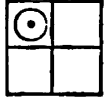
Similarly we may represent the three similar Propositions “Some x are y' ”, “Some x' are y ”, and “Some x' are y' ”.

[The Reader should make out all these for himself.

In the “books” example, these three Propositions would be “Some old books are foreign”, &c.]

Let us take, next, the Proposition “Some y are x ”.

This tells us that at least *one* Thing, in the *West* Half, is also in the *North* Half. Hence it must be in the space *common* to them, that is, in the *North-West Cell*. Hence the North-West Cell is *occupied*. And this we can represent by placing a *Red Counter* in it.



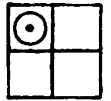
[In the “books” example, this Proposition would be “Some English books are old”.]

Similarly we may represent the three similar Propositions “Some y are x' ”, “Some y' are x ”, and “Some y' are x' ”.

[The Reader should make out all these for himself.

In the “books” example, these three Propositions would be “Some English books are new”, &c.]

We see that this *one* Diagram has now served to represent no less than *three* Propositions, viz.



- (1) “Some xy exist ;
- (2) Some x are y ;
- (3) Some y are x ”.

Hence these three Propositions are equivalent.

[In the “books” example, these Propositions would be

- (1) “Some old English books exist” ;
- (2) “Some old books are English” ;
- (3) “Some English books are old”.]

The two equivalent Propositions, “Some x are y ” and “Some y are x ”, are said to be ‘**Converse**’ to each other ; and the Process, of changing one into the other, is called ‘**Converting**’, or ‘**Conversion**’.

[For example, if we were told to convert the Proposition “Some apples are not ripe,”

we should first choose our Univ. (say “fruit”), and then complete the Proposition, by supplying the Substantive “fruit” in the Predicate, so that it would be

“Some apples are not-ripe fruit” ;

and we should then convert it by interchanging its Terms, so *that it would be*

“Some not-ripe fruit are apples”.]

Similarly we may represent the three similar Trios of equivalent Propositions; the whole Set of *four* Trios being as follows:—

(1) “Some xy exist” = “Some x are y ” = “Some y are x ”.

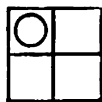
(2) “Some xy' exist” = “Some x are y' ” = “Some y' are x ”.

(3) “Some $x'y$ exist” = “Some x' are y ” = “Some y are x' ”.

(4) “Some $x'y'$ exist” = “Some x' are y' ” = “Some y' are x' ”.

Let us take, next, the Proposition “No x are y ”.

This tells us that no Thing, in the *North* Half, is also in the *West* Half. Hence there is *nothing* in the space *common* to them, that is, in the *North-West Cell*. Hence the North-West Cell is *empty*. And this we can represent by placing a *Grey* Counter in it.



[In the “books” example, this Proposition would be “No old books are English”.]

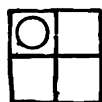
Similarly we may represent the three similar Propositions “No x are y' ”, and “No x' are y ”, and “No x' are y' ”.

[The Reader should make out all these for himself.

In the “books” example, these three Propositions would be “No old books are foreign”, &c.]

Let us take, next, the Proposition “No y are x ”.

This tells us that no Thing, in the *West* Half, is also in the *North* Half. Hence there is *nothing* in the space *common* to them, that is, in the *North-West Cell*. That is, the North-West Cell is *empty*. And this we can represent by placing a *Grey* Counter in it.



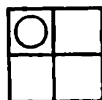
[In the “books” example, this Proposition would be “No English books are old”.]

Similarly we may represent the three similar Propositions “No y are x' ”, “No y' are x ”, and “No y' are x' ”.

[The Reader should make out all these for himself.

In the “books” example, these three Propositions would be “No English books are new”, &c.]

We see that this *one* Diagram has now served to represent no less than *three* Propositions, viz.



- (1) "No xy exist ;
- (2) No x are y ;
- (3) No y are x ."

Hence these three Propositions are equivalent.

[In the "books" example, these Propositions would be

- (1) "No old English books exist ;
- (2) No old books are English ;
- (3) No English books are old".]

The two equivalent Propositions, "No x are y " and "No y are x ", are said to be 'Converse' to each other.

[For example, if we were told to convert the Proposition

"No porcupines are talkative",
we should first choose our Univ. (say "animals"), and then complete the Proposition, by supplying the Substantive "animals" in the Predicate, so that it would be

"No porcupines are talkative animals",
and we should then convert it, by interchanging its Terms, so that it would be

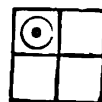
"No talkative animals are porcupines".]

Similarly we may represent the three similar Trios of equivalent Propositions ; the whole Set of *four* Trios being as follows :—

- (1) "No xy exist" = "No x are y " = "No y are x ".
- (2) "No xy' exist" = "No x are y' " = "No y' are x ".
- (3) "No $x'y$ exist" = "No x' are y " = "No y are x' ".
- (4) "No $x'y'$ exist" = "No x' are y' " = "No y' are x' ".

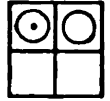
Let us take, next, the Proposition "All x are y ".

This clearly tells us that "Some x are y ". So we may at any rate place a *Red Counter* in the *North-West Cell*.



But the Proposition tells us *more* than this. It tells us, not only that at least *one* Thing, in the North Half, is in the North-West Cell, but also that *all* Things, in the North Half, are in the North-West Cell. Hence there are none of them

in the North-East Cell ; that is, the North-East Cell is *empty*. And *this* we can of course represent by placing a *Grey* Counter in it.



[Note that the *Subject* of the given Proposition settles which *Half* we are to use ; and that its *Predicate* settles in which *portion* of that *Half* we are to place the Red Counter.]

Similarly we may represent the seven similar Propositions “All x are y ”, “All x' are y ”, “All x' are y' ”, “All y are x ”, “All y are x' ”, “All y' are x ”, and “All y' are x' ”.

[The Reader should make out all these for himself.]

We see then that “All x are y ” is a *Double* Proposition ; and that it is equivalent to the *two* Propositions

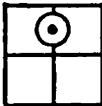
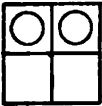
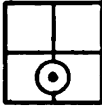
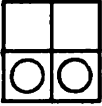
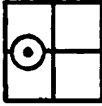
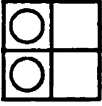
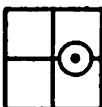
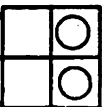
- (1) “Some x are y ;
- (2) No x are y' ”.

[In the “books” example, this Proposition would be “All old books are English”.

Hence this is a *Double* Proposition, and is equivalent to the *two* Propositions

- (1) “Some old books are English ;
- (2) No old books are foreign ”.]

TABLE II.

Some x exist		No x exist	
Some x' exist		No x' exist	
Some y exist		No y exist	
Some y' exist		No y' exist	

The Reader should now get his genial friend to question him, severely, on these two Tables.

The *Inquisitor* should have the Tables before him: but the *Victim* should have nothing but a blank Diagram, like this, and the Counters, with which he is to represent the various propositions named by his friend, e.g. "Some y exist", "No y' are x ", "All x are y ", "No $x'y$ exist", "Some x' are y ", &c. &c.

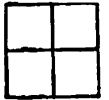


TABLE III.

Some xy exist = Some x are y = Some y are x		All x are y	
Some xy' exist = Some x are y' = Some y' are x		All x are y'	
Some $x'y$ exist = Some x' are y = Some y are x'		All x' are y	
Some $x'y'$ exist = Some x' are y' = Some y' are x'		All x' are y'	
No xy exist = No x are y = No y are x		All y are x	
No xy' exist = No x are y' = No y' are x		All y are x'	
No $x'y$ exist = No x' are y = No y are x'		All y' are x	
No $x'y'$ exist = No x' are y' = No y' are x'		All y' are x'	
Some x are y , and some are y'		Some y are x , and some are x'	
Some x' are y , and some are y'		Some y' are x , and some are x'	

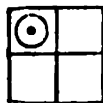
CHAPTER IV.

INTERPRETATION OF BILITERAL DIAGRAM, WHEN MARKED WITH COUNTERS.

THE Diagram is supposed to be set before us, with certain Counters placed upon it ; and the problem is to find out what Proposition, or Propositions, the Counters represent.

As the process is simply the reverse of that discussed in the previous Chapter, we can avail ourselves of the results there obtained, as far as they go.

First, let us suppose that we find a *Red* Counter placed in the North-West Cell.



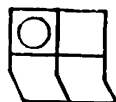
We know that this represents each of the Trio of equivalent Propositions

“Some xy exist” = “Some x are y ” = “Some y are x ”.

Similarly we may interpret a *Red* Counter, when placed in the North-East, or South-West, or South-East Cell.

[The Reader should make out all these for himself.]

Next, let us suppose that we find a Grey Counter placed in the North-West Cell.



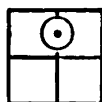
We know that this represents each of the Trio of equivalent Propositions

“No xy exist” = “No x are y ” = “No y are x ”.

Similarly we may interpret a *Grey* Counter, when placed in the North-East, or South-West, or South-East Cell.

[The Reader should make out all these for himself.]

Next, let us suppose that we find a *Red* Counter placed on the partition which divides the North Half.

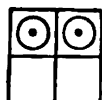


We know that this represents the Proposition “Some x exist.”

Similarly we may interpret a *Red* Counter, when placed on the partition which divides the South, or West, or East Half.

[The Reader should make out all these for himself.]

Next, let us suppose that we find *two Red* Counters placed in the North Half, one in each Cell.

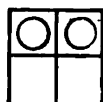


This is an arrangement that we have not yet considered. However, it is easy to see that it represents the *Double* Proposition “Some x are y and some are y' ”.

Similarly we may interpret *two Red* Counters, when placed in the South, or West, or East Half.

[The Reader should make out all these for himself.]

Next, let us suppose that we find *two Grey* Counters placed in the North Half, one in each Cell.

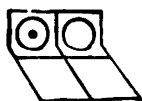


We know that this represents the Proposition “No x exist”.

Similarly we may interpret *two Grey* Counters, when placed in the South, or West, or East Half.

[The Reader should make out all these for himself.]

Lastly, let us suppose that we find a *Red* and a *Grey* Counter placed in the North Half, the *Red* in the North-West Cell, and the *Grey* in the North-East Cell.



We know that this represents the Proposition, "All x are y ".

[Note that the *Half*, occupied by the two Counters, settles what is to be the *Subject* of the Proposition, and that the *Cell*, occupied by the *Red* Counter, settles what is to be its *Predicate*.]

Similarly we may interpret a *Red* and a *Grey* Counter, when placed in any one of the seven similar positions

Red in North-East, *Grey* in North-West ;

Red in South-West, *Grey* in South-East ;

&c.

[The Reader should make out all these for himself.]

Once more the genial friend must be appealed to, and requested to examine the Reader on Tables II and III, and to make him not only *represent* Propositions, but also *interpret* Diagrams when marked with Counters.

The Questions and Answers should be like this:—

Q. Represent "No x' are y' ."

A. Grey Counter in S.E. Cell.

Q. Interpret Red Counter on E. partition.

A. "Some y' exist."

Q. Represent "All y' are x ."

A. Red in N.E. Cell ; Grey in S.E.

Q. Interpret Grey in S.W. Cell.

A. "No $x'y$ exist" = "No x' are y " = "No y are x' ".

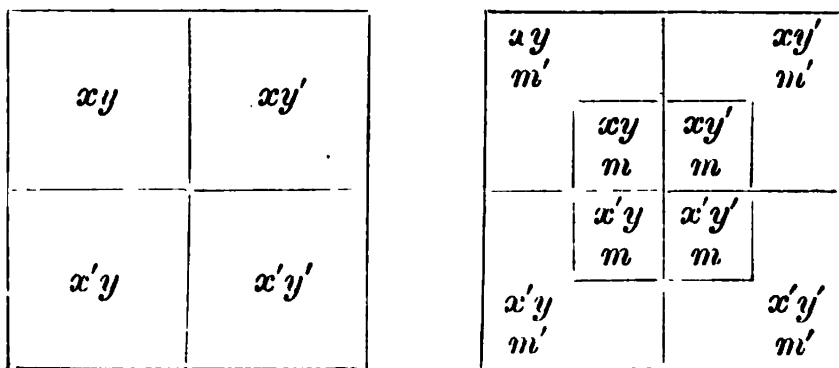
&c., &c.

At first the Examinee will need to have the Board and Counters before him ; but he will soon learn to dispense with these, and to answer with his eyes shut, or gazing into vacancy.

[Work Examples § 1, 5—8 (p. 97).]

BOOK IV.

THE TRILITERAL DIAGRAM.



CHAPTER I.

SYMBOLS AND CELLS.

FIRST, let us suppose that the above *left-hand* Diagram is the *Biliteral* Diagram that we have been using in Book III., and that we change it into a *Triliteral* Diagram by drawing an *Inner Square*, so as to divide each of its 4 Cells into 2 portions, thus making 8 Cells altogether. The *right-hand* Diagram shows the result.

[The Reader is strongly advised, in reading this Chapter, *not* to refer to the above Diagrams, but to make a large copy of the right-hand one for himself, *without any letters*, and to have it by him while he reads, and keep his finger on that particular *part* of it, about which he is reading.]

Secondly, let us suppose that we have selected a certain Attribute (or Set of Attributes regarded as a single Attribute), which we may call " m ", and have subdivided the xy -Class into the two Classes whose peculiar Attributes are m and m' , and that we have assigned the N.W. *Inner* Cell to the one (which we may call "the Class of xym -Things", or "the xym -Class"), and the N.W. *Outer* Cell to the other (which we may call "the Class of xym' -Things", or "the xym' -Class").

[Thus, in the "books" example, we might say "Let m mean 'bound', so that m' will mean 'unbound'", and we might suppose that we had subdivided the Class "old English books" into the two Classes, "old English bound books" and "old English unbound books", and had assigned the N.W. *Inner* Cell to the one, and the N.W. *Outer* Cell to the other.]

Thirdly, let us suppose that we have subdivided the xy' -Class, the $x'y$ -Class, and the $x'y'$ -Class in the same manner, and have, in each case, assigned the *Inner* Cell to the Class possessing the Attribute m , and the *Outer* Cell to the Class possessing the Attribute m' .

[Thus, in the "books" example, we might suppose that we had subdivided the "new English books" into the two Classes, "new English bound books" and "new English unbound books", and had assigned the S.W. *Inner* Cell to the one, and the S.W. *Outer* Cell to the other.]

It is evident that we have now assigned the *Inner Square* to the m -Class, and the *Outer Border* to the m' -Class.

[Thus, in the "books" example, we have assigned the *Inner Square* to "bound books" and the *Outer Border* to "unbound books".]

When the Reader has made himself familiar with this Diagram, he ought to be able to find, in a moment, the Compartment assigned to a particular *pair* of Attributes, or the Cell assigned to a particular *trio* of Attributes. The following Rules will help him in doing this:—

(1) Arrange the Attributes in the order x, y, m .

(2) Take the *first* of them and find the Compartment assigned to it.

(3) Then take the *second*, and find what *portion* of that Compartment is assigned to it.

(4) Treat the *third*, if there is one, in the same way.

[For example, suppose we have to find the Compartment assigned to ym . We say to ourselves " y has the *West* Half; and m has the *Inner* portion of that West Half."

Again, suppose we have to find the Cell assigned to $x'ym'$. We say to ourselves " x' has the *South* Half; y has the *West* portion of that South Half, i.e. has the *South-West Quarter*; and m' has the *Outer* portion of that South-West Quarter."]

The Reader should now get his genial friend to question him on the Table given on the next page, in the style of the following specimen-Dialogue.

Q. Attributes for South Half, Inner Portion?

A. $x'm$.

Q. Compartment for m' ?

A. The Outer Border.

Q. Attributes for North-East Quarter, Outer Portion

A. $xy'm'$.

Q. Compartment for ym ?

A. West Half, Inner Portion.

Q. Attribute for South Half?

A. x' .

Q. Compartment for $x'y'm$?

A. South-East Quarter, Inner Portion.

&c. &c.

TABLE IV.

<i>Attributes of Classes.</i>	<i>Compartments, or Cells, assigned to them.</i>
x	North Half.
x'	South „
y	West „
y'	East „
m	Inner Square.
m'	Outer Border.
xy	North-West Quarter.
xy'	„ East „
$x'y$	South-West „
$x'y'$	„ East „
xm	North Half, Inner Portion.
xm'	„ „ Outer „
$x'm$	South „ Inner „
$x'm'$	„ „ Outer „
ym	West „ Inner „
ym'	„ „ Outer „
$y'm$	East „ Inner „
$y'm'$	„ „ Outer „
xym	North-West Quarter, Inner Portion.
xym'	„ „ „ Outer „
$xy'm$	„ East „ Inner „
$xy'm'$	„ „ „ Outer „
$x'ym$	South-West „ Inner „
$x'ym'$	„ „ „ Outer „
$x'y'm$	„ East „ Inner „
$x'y'm'$	„ „ „ Outer „

CHAPTER II.

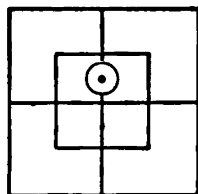
REPRESENTATION OF PROPOSITIONS.

§ 1.

Representation of Propositions of Existence.

LET us take, first, the Proposition “Some xm exist”.

This tells us that there is at least *one* Thing in the Inner portion of the North Half; that is, that this Compartment is *occupied*. And this we can evidently represent by placing a *Red Counter* on the partition which divides it.



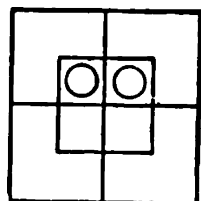
[In the “books” example, this Proposition would mean “Some old bound books exist” (or “There are some old bound books”).]

Similarly we may represent the seven similar Propositions, containing m or m' , viz. “Some xm' exist”, “Some $x'm$ exist”, “Some $x'm'$ exist”, “Some ym exist”, “Some ym' exist”, “Some $y'm$ exist”, and “Some $y'm'$ exist”.

[The Reader should make out all these for himself.]

Let us take, next, the Proposition “No xm exist”.

This tells us that there is *nothing* in the Inner portion of the North Half; that is, that this Compartment is *empty*. And this we can represent by placing *two Grey Counters* in it, one in each Cell.



Similarly we may represent the seven similar Propositions, containing m or m' , viz. "No xm' exist", "No $x'm$ exist", &c.

[The Reader should make out all these for himself.]

These sixteen Propositions of Existence are the only ones that we shall have to represent on this Diagram.

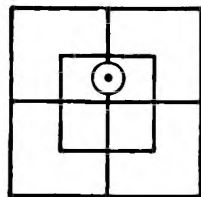
§ 2.

Representation of Propositions of Relation.

Let us take, first, the Pair of Converse (and therefore equivalent) Propositions

"Some x are m " = "Some m are x ."

We know that each of these is equivalent to the Proposition of Existence "Some xm exist", which we already know how to represent.



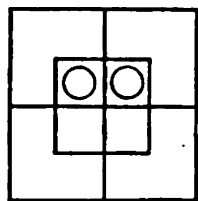
Similarly for the seven similar Pairs, containing m or m' .

[The Reader had better make out all these for himself.]

Let us take, next, the Pair of Converse (and therefore equivalent) Propositions

"No x are m " = "No m are x ."

We know that each of these is equivalent to the Proposition of Existence "No xm exist", which we already know how to represent.

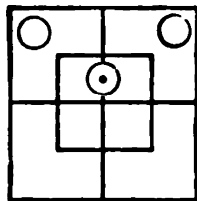


Similarly for the seven similar Pairs, containing m or m' .

[The Reader had better make out all these for himself.]

Let us take, next, the Proposition “All x are m .”

We know that this is a *Double* Proposition, and equivalent to the *two* Propositions “Some x are m ” and “No x are m' ”, both of which we already know how to represent.



Similarly for the fifteen similar Propositions, containing m or m' .

[The Reader had better make out all these for himself.]

These thirty-two Propositions of Relation are the only ones that we shall have to represent on this Diagram.

The Reader should now get his genial friend to question him on the following four Tables, making occasional excursions into the earlier Tables, and dodging about, from Table to Table, as long as his Victim's temper will stand it.

The Victim should have nothing before him but 2 blank Diagrams (Bilateral and Trilateral), 2 Red Counters, and 2 Grey ones, with which he is to represent the various Propositions named by the Inquisitor, *e.g.*

(from Table VI.) “Represent, on Trilateral Diagram, ‘No y' are m' ’.”

(from Table III.) “Represent, on Bilateral Diagram, ‘All x' are y' ’.”

(from Table V.) “Represent, on Trilateral Diagram, ‘Some xm' exist’.”

&c. &c.

TABLE V.

	<p>Some xm exist = Some x are m = Some m are x</p>	
<p>No xm exist = No x are m = No m are x</p>		
	<p>Some xm' exist = Some x are m' = Some m' are x</p>	
<p>No xm' exist = No x are m' = No m' are x</p>		
	<p>Some $x'm$ exist = Some x' are m = Some m are x'</p>	
<p>No $x'm$ exist = No x' are m = No m are x'</p>		
	<p>Some $x'm'$ exist = Some x' are m' = Some m' are x'</p>	
<p>No $x'm'$ exist = No x' are m' = No m' are x'</p>		

TABLE VI.

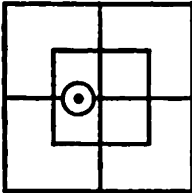
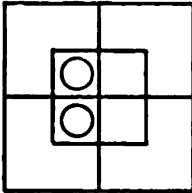
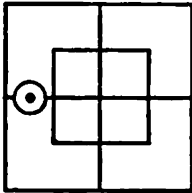
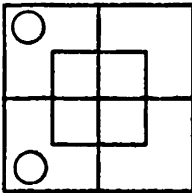
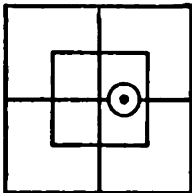
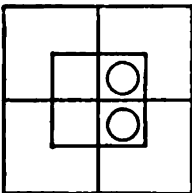
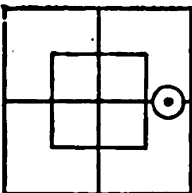
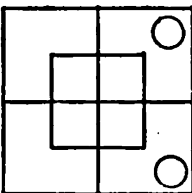
	<p>Some ym exist $=$ Some y are m $=$ Some m are y</p>	
	<p>Some ym' exist $=$ Some y are m' $=$ Some m' are y</p>	
	<p>Some $y'm$ exist $=$ Some y' are m $=$ Some m are y'</p>	
	<p>Some $y'm'$ exist $=$ Some y' are m' $=$ Some m' are y'</p>	
	<p>No $y'm'$ exist $=$ No y' are m' $=$ No m' are y'</p>	

TABLE VII.

	All x are m	
	All x are m'	
	All x' are m	
	All x' are m'	
	All m are x	
	All m are x'	
	All m' are x	
	All m' are x'	

TABLE VIII.

	All y are m	
	All y' are m	
	All m are y	
	All m' are y	

CHAPTER III.

REPRESENTATION OF TWO PROPOSITIONS OF RELATION ON THE SAME DIAGRAM.

THE Reader had better now begin to draw little Diagrams for himself, and to mark them with the Digits "I" and "O", instead of using the Board and Counters: he may put a "I" to represent a *Red* Counter (this may be interpreted to mean "There is at least *one* Thing here"), and a "O" to represent a *Grey* Counter (this may be interpreted to mean "There is *nothing* here").

The Pair of Propositions, that we shall have to represent, will always be, one in terms of x and m , and the other in terms of y and m .

When we have to represent a Proposition beginning with "All", we break it up into the *two* Propositions to which it is equivalent.

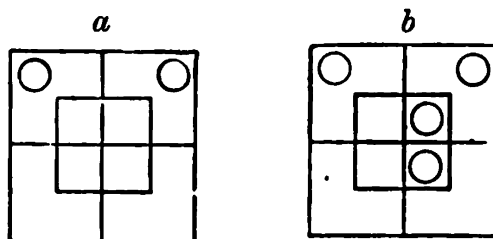
When we have to represent, on the same Diagram, Propositions, of which some begin with "Some" and others with "No", we represent the *negative* ones *first*. This will sometimes save us from having to put a "I" "on a fence" and afterwards having to shift it into a Cell.

[Let us work a few examples.

(1)
"No x are m' ;
No y' are m ".

Let us first represent "No x are m' ". This gives us Diagram *a*.

Then, representing "No y' are m " on the same Diagram, we get Diagram *b*.



(2)

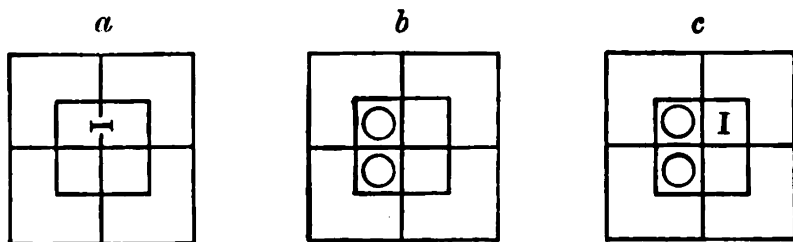
“Some m are x ,
No m are y ”.

If, neglecting the Rule, we were to begin with “Some m are x ”, we should get Diagram *a*.

And if we were then to take “No m are y ”, which tells us that the Inner N.W. Cell is *empty*, we should be obliged to take the “I” off the fence (as it no longer has the choice of *two* Cells), and to put it into the Inner N.E. Cell, as in Diagram *c*.

This trouble may be saved by beginning with “No m are y ”, as in Diagram *b*.

And *now*, when we take “Some m are x ”, there is no fence to sit on! The “I” has to go, at once, into the N.E. Cell, as in Diagram *c*.



(3)

“No x' are m' ;
All m are y ”.

Here we begin by breaking up the Second into the two Propositions to which it is equivalent. Thus we have *three* Propositions to represent, viz.—

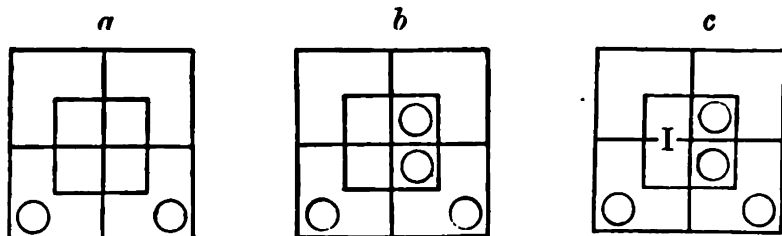
- (1) “No x' are m' ;
- (2) Some m are y ;
- (3) No m are y' ”.

These we will take in the order 1, 3, 2.

First we take No. (1), viz. “No x' are m' ”. This gives us Diagram *a*.

Adding, to this, No. (3), viz. "No. m are y ", we get Diagram *b*.

This time the "I", representing No. (2), viz. "Some m are y ," has to sit on the fence, as there is no "O" to order it off! This gives us Diagram *c*.



(4)
"All m are x ;
All y are m ".

Here we break up *both* Propositions, and thus get *four* to represent, viz.—

- (1) "Some m are x ;
- (2) No m are x' ;
- (3) Some y are m ;
- (4) No y are m' .

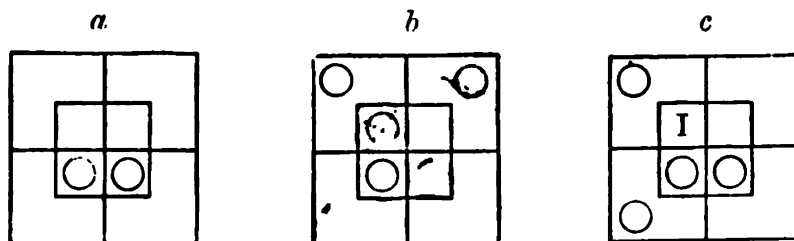
These we will take in the order 2, 4, 1, 3.

First we take No. (2), viz. "No m are x' ". This gives us Diagram *a*.

To this we add No. (4), viz. "No y are m' ", and thus get Diagram *b*.

If we were to add to this No. (1), viz. "Some m are x ", we should have to put the "I" on a fence: so let us try No. (3) instead, viz. "Some y are m ". This gives us Diagram *c*.

And now there is no need to trouble about No. (1), as it would not add anything to our information to put a "I" on the fence. The Diagram *already* tells us that "Some m are x ".]



[Work Examples § 1, 9—12 (p. 97) ; § 2, 1—20 (p. 98.)]

CHAPTER IV.

INTERPRETATION, IN TERMS OF x AND y , OF TRILITERAL DIAGRAM, WHEN MARKED WITH COUNTERS OR DIGITS.

THE problem before us is, given a marked Triliteral Diagram, to ascertain *what* Propositions of Relation, in terms of x and y , are represented on it.

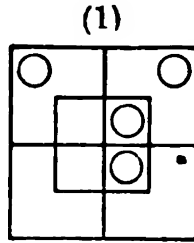
The best plan, for a *beginner*, is to draw a *Biliteral* Diagram alongside of it, and to transfer, from the one to the other, all the information he can. He can then read off, from the Biliteral Diagram, the required Propositions. After a little practice, he will be able to dispense with the Biliteral Diagram, and to read off the result from the Triliteral Diagram itself.

To *transfer* the information, observe the following Rules:—

- (1) Examine the N.W. Quarter of the Triliteral Diagram.
- (2) If it contains a “I”, in *either* Cell, it is certainly *occupied*, and you may mark the N.W. Quarter of the Biliteral Diagram with a “I”.
- (3) If it contains *two* “O”s, one in *each* Cell, it is certainly *empty*, and you may mark the N.W. Quarter of the Biliteral Diagram with a “O”.

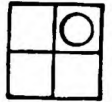
- (4) Deal in the same way with the N.E., the S.W., and the S.E. Quarter.

[Let us take, as examples, the results of the four Examples worked in the previous Chapters.



In the N.W. Quarter, only *one* of the two Cells is marked as *empty*: so we do not know whether the N.W. Quarter of the Biliteral Diagram is *occupied* or *empty*: so we cannot mark it.

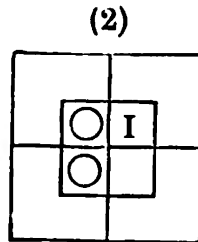
In the N.E. Quarter, we find *two* "O"s: so *this* Quarter is certainly *empty*; and we mark it so on the Biliteral Diagram.



In the S.W. Quarter, we have no information *at all*.

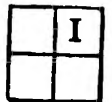
In the S.E. Quarter, we have not enough to use.

We may read off the result as "No *x* are *y*'", or "No *y*' are *x*," whichever we prefer.



In the N.W. Quarter, we have not enough information to use.

In the N.E. Quarter, we find a "I". This shows us that it is *occupied*: so we may mark the N.E. Quarter on the Biliteral Diagram with a "I".

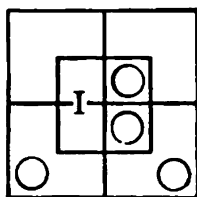


In the S.W. Quarter, we have not enough information to use.

In the S.E. Quarter, we have none at all.

We may read off the result as "Some *x* are *y*'", or "Some *y*' are *x*", whichever we prefer.

(3)

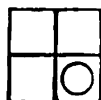


In the N. W. Quarter, we have *no* information. (The "I", sitting on the fence, is of no use to us until we know on *which* side he means to jump down !)

In the N. E. Quarter, we have not enough information to use.

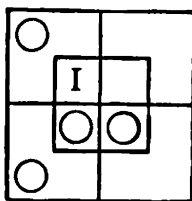
Neither have we in the S. W. Quarter.

The S. E. Quarter is the only one that yields enough information to use. It is certainly *empty*: so we mark it as such on the Biliteral Diagram.



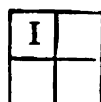
We may read off the result as "No x' are y' ", or "No y' are x' ", whichever we prefer.

(4)

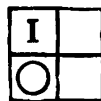


The N. W. Quarter is *occupied*, in spite of the "O" in the Outer Cell. So we mark it with a "I" on the Biliteral Diagram.

The N. E. Quarter yields no information.



The S. W. Quarter is certainly *empty*. So we mark it as such on the Biliteral Diagram.



The S. E. Quarter does not yield enough information to use.

We read off the result as "All y are x ."]

[Review Tables V, VI. (pp. 46, 47). Work Examples § 1, 13—16 (p. 97); § 2, 21—32 (p. 98); § 3, 1—20 (p. 99)].

BOOK V.

SYLLOGISMS.

CHAPTER I.

INTRODUCTORY.

WHEN a Set of Things is divided by the Process of *Dichotomy by Contradiction*, each of the two Classes, thus obtained, may be said to be '**codivisional**' with itself and with the other.

[Note that they are so called, because they are obtained by one and the same Process of *Division*.

Similarly, if several events happen at one and the same *time*, each of them may be said to be "contemporaneous" with itself and with the others.]

When a Trio of Biliteral Propositions of Relation is such that

- (1) every two of them contain between them a pair of codivisional Classes,
 - (2) the three Propositions are so related that, if the first two were true, the third would be true,
- the Trio is called a 'Syllogism'; the first two are called its 'Premisses', and the third its 'Conclusion'; also the pair*

of codivisional Terms in the Premisses may be called its '**Eliminands**', and the other two its '**Retinends**'.

The Conclusion of a Syllogism is said to be '**consequent**' from its Premisses: hence it is usual to prefix to it the word "Therefore" (or the Symbol " \therefore ").

[Note that the '**Eliminands**' are so called because they are *eliminated*, and do not appear in the Conclusion; and that the '**Retinends**' are so called because they are *retained*, and *do* appear in the Conclusion.

Note also that the question, whether the Conclusion is or is not *consequent* from the Premisses, is not affected by the *actual* truth or falsity of any of the Trio, but depends entirely on their *relation to each other*.

As a specimen-Syllogism, let us take the Trio

"No x -Things are m -Things;
No y -Things are m' -Things.
No x -Things are y -Things."

which we may write, as explained at p. 26, thus:—

"No x are m ;
No y are m' .
No x are y ".

Here the first and second contain the pair of codivisional Classes m and m' ; the first and third contain the pair x and x ; and the second and third contain the pair y and y .

Also the three Propositions are so related that, if the first two were true, the third would be true.

Hence the Trio is a *Syllogism*; the two Propositions, "No x are m ", and "No y are m' ", are its *Premisses*; the Proposition "No x are y " is its *Conclusion*; the Terms m and m' are its *Eliminands*; and the Terms x and y are its *Survivors*.

Hence we may write it thus:—

"No x are m ;
No y are m' .
 \therefore No x are y ".

As a second specimen, let us take the Trio

"All cats understand French;
Some chickens are cats.
Some chickens understand French".

These, put into normal form, are

"All cats are creatures understanding French;
Some chickens are cats.
Some chickens are creatures understanding French"

Here the first and second contain the pair of codivisional Classes “cats” and “cats”; the first and third contain the pair “creatures understanding French” and “creatures understanding French”; and the second and third contain the pair “chickens” and “chickens”.

Also the three Propositions are (as we shall see at p. 64) so related that, if the first two were true, the third would be true. (The first two are, as it happens, *not* strictly true in *our* planet. But there is nothing to hinder them from being true in some *other* planet, say *Mars* or *Jupiter*—in which case the third would *also* be true in that planet, and its inhabitants would probably engage chickens as nursery-governesses !)

Hence the Trio is a *Syllogism*; the two Propositions, “All cats understand French” and “Some chickens are cats”, are its *Premisses*; the Proposition “Some chickens understand French” is its *Conclusion*; the Terms “cats” and “cats” are its *Eliminands*; and the Terms, “creatures understanding French” and “chickens,” are its *Retinends*.

Hence we may write it thus:—

“All cats understand French ;
Some chickens are cats ;
∴ Some chickens understand French”.]

CHAPTER II.

PROBLEMS IN SYLLOGISMS.

§ 1.

Introductory.

WHEN the Terms of a Proposition are represented by *words*, it is said to be '**concrete**'; when by *letters*, '**abstract**.'

To translate a Proposition from concrete into abstract form, we regard each Term as a *Species* of the 'Univ.,' and we choose a letter to represent its *peculiar Attribute*.

[For example, suppose we wish to translate "Some soldiers are brave" into abstract form. We may take "men" as Univ., and regard "soldiers" and "brave men" as *Species* of the *Genus* "men"; and we may choose x to represent the peculiar Attribute (say "military") of "soldiers," and y to represent "brave." Then the Proposition may be written "Some military men are brave men"; *i.e.* "Some x -men are y -men"; *i.e.* (omitting "men," as explained at p. 26) "Some x are y ."

In practice, we should merely say "Let x = soldiers, y = brave," and at once translate "Some soldiers are brave" into "Some x are y ."]

The Problems we shall have to solve are of two kinds, viz.

(1) "Given a Pair of Propositions of Relation, which contain between them a pair of codivisional Classes, and which are proposed as Premisses: to ascertain what Conclusion, if any, is consequent from them."

(2) "Given a Trio of Propositions of Relation, of which every two contain a pair of codivisional Classes, and which are proposed as a Syllogism: to ascertain whether the proposed Conclusion is consequent from the proposed Premisses, and, if so, whether it is complete."

These Problems we will discuss separately.

§ 2

Given a Pair of Propositions of Relation, which contain between them a pair of codivisional Classes, and which are proposed as Premisses: to ascertain what Conclusion, if any, is consequent from them.

The Rules, for doing this, are as follows:—

- (1) Name the 'Universe of Discourse'.
- (2) Construct a Dictionary, making m and m (or m and m') represent the pair of codivisional Classes, and x (or x') and y (or y') the other two.
- (3) Translate the proposed Premisses into abstract form.
- (4) Represent them, together, on a Triliteral Diagram.
- (5) Ascertain what Proposition, if any, in terms of x and y , is *also* represented on it.
- (6) Translate this into concrete form.

It is evident that, if the proposed Premisses were true, this other Proposition would *also* be true. Hence it is a *Conclusion* consequent from the proposed Premisses.

[Let us work some examples.

(1)

"No son of mine is dishonest ;
People always treat an honest man with respect".

Taking "men" as Univ., we may write these as follows:—

"No sons of mine are dishonest men ;
All honest men are men treated with respect".

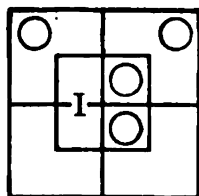
We can now construct our Dictionary, viz. m = honest ;
 x = sons of mine ; y = treated with respect.

(Note that the expression " x = sons of mine" is an abbreviated form of " x = the Attribute (or Set of Attributes) peculiar to 'sons of mine', when regarded as a Species of 'men'".)

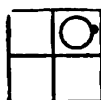
The next thing is to translate the proposed Premisses into abstract form, as follows:—

"No x are m' ;
All m are y ".

Next, by the process described at p. 50, we represent these on a Trilateral Diagram, thus :—



Next, by the process described at p. 53, we transfer to a Bilateral Diagram all the information we can.



The result we can read either as “No x are y' ” or as “No y are x ,” whichever we prefer. So we refer to our Dictionary, to see which will look best ; and we choose

“No x are y' ”,

which, translated into concrete form, is

“No son of mine ever fails to be treated with respect.”

(2)

“All cats understand French ;
Some chickens are cats”.

Taking “creatures” as Univ., we may write these as follows :—

“All cats are creatures understanding French ;
Some chickens are cats”.

We can now construct our Dictionary, viz. m = cats ;
 x = understanding French ; y = chickens.

The proposed Premisses, translated into abstract form, are

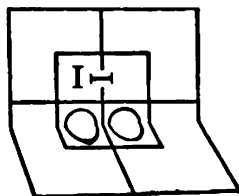
“All m are x ;
Some y are m ”.

In order to represent these on a Trilateral Diagram, we break up the first into the two Propositions to which it is equivalent, and thus get the *three* Propositions

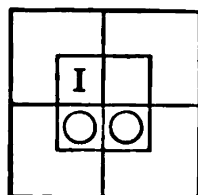
- (1) “Some m are x ;
- (2) No m are x' ;
- (3) Some y are m ”.

The Rule, given at p. 50, would make us take these in the order 2, 1, 3.

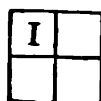
This, however, would produce the result



So it would be better to take them in the order 2, 3, 1. Nos. (2) and (3) give us the result here shown; and now we need not trouble about No. (1), as the Proposition "Some m are x " is *already* represented on the Diagram.



Transferring our information to a Biliteral Diagram, we get



This result we can read either as "Some x are y " or "Some y are x ".

After consulting our Dictionary, we choose

"Some y are x ",

which, translated into concrete form, is

"Some chickens understand French."

(3)

"All diligent students are successful ;
All ignorant students are unsuccessful".

Let Univ. be "students"; m = successful; x = diligent ;
 y = ignorant.

These Premisses, in abstract form, are

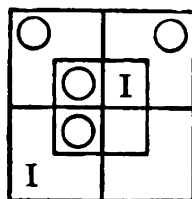
"All x are m ;
All y are m' ".

These, broken up, give us the four Propositions

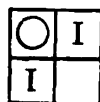
- (1) "Some x are m ;
- (2) No x are m' ;
- (3) Some y are m' ;
- (4) No y are m ".

which we will take in the order 2, 4, 1, 3.

Representing these on a Triliteral Diagram, we get



And this information, transferred to a Biliteral Diagram, is



Here we get *two* Conclusions, viz.

"All x are y' ;
All y are x' ".

And these, translated into concrete form, are

“ All diligent students are (not-ignorant, i.e.) learned ;
 All ignorant students are (not-diligent, i.e.) idle ”.
 (See p. 4.)

(4)

“ Of the prisoners who were put on their trial at the last Assizes, all, against whom the verdict ‘ guilty ’ was returned, were sentenced to imprisonment ;
 Some, who were sentenced to imprisonment, were also sentenced to hard labour ”.

Let Univ. be “ the prisoners who were put on their trial at the last Assizes ” ; m = who were sentenced to imprisonment ; x = against whom the verdict ‘ guilty ’ was returned ; y = who were sentenced to hard labour.

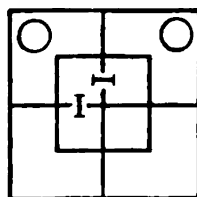
The Premisses, translated into abstract form, are

“ All x are m ;
 Some m are y ”.

Breaking up the first, we get the three

- (1) “ Some x are m ;
- (2) No x are m' ;
- (3) Some m are y ”.

Representing these, in the order 2, 1, 3, on a Trilateral Diagram, we get



Here we get no Conclusion at all.

The Reader would very likely have guessed, if he had seen *only* the Premisses, that the Conclusion would be

“ Some, against whom the verdict ‘ guilty ’ was returned, were sentenced to hard labour ”.

Let him play the trick on some innocent friend, not so good a Logician as himself, by showing him the Premisses and the above Conclusion, and asking him if he thinks “ it’s all right ”. That innocent friend will reply “ Why, of *course* it is ! It needs no Logic to see *that* ! ”]

I will now work out, in their briefest form, as models for the Reader to imitate in working examples, the above four *concrete Problems*.

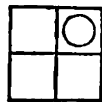
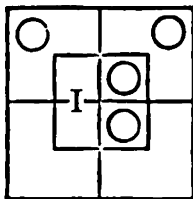
(1) [see p. 60]

“No son of mine is dishonest ;

People always treat an honest man with respect.”

Univ. “men” ; m = honest ; x = my sons ; y = treated with respect.

“No x are m' ;
All m are y .”



\therefore No x are y' ;

i.e. “No son of mine ever fails to be treated with respect.”

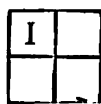
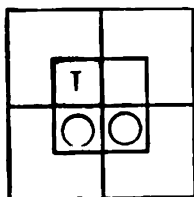
(2) [see p. 61]

“All cats understand French ;

Some chickens are cats ”.

Univ. “creatures” ; m = cats ; x = understanding French ; y = chickens.

“All m are x ;
Some y are m .”



\therefore “Some y are x .”

i.e. “Some chickens understand French.”

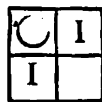
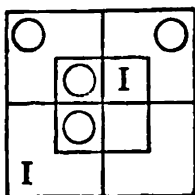
(3) [see p. 62]

“All diligent students are successful ;

All ignorant students are unsuccessful ”.

Univ. “students” ; m = successful ; x = diligent ; y = ignorant.

“All x are m ;
All y are m' .”



\therefore “All x are y' ;
All y are x' ”

i.e. All diligent students are learned ; and all ignorant students are idle ”.

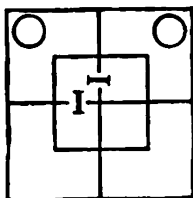
(4) [see p. 63]

“Of the prisoners who were put on their trial at the last Assizes, all, against whom the verdict ‘guilty’ was returned, were sentenced to imprisonment;

Some, who were sentenced to imprisonment, were also sentenced to hard labour”.

Univ. “prisoners who were &c.”; m = sentenced to imprisonment; x = against whom the verdict ‘guilty’ was returned; y = sentenced to hard labour.

“All x are m ;
Some m are y .”



There is no
Conclusion.

[Review Tables VII, VIII (pp. 48, 49). Work Examples § 1, 17—21 (p. 97); § 4, 1—6 (p. 100); § 5, 1—6 (p. 101).]

§ 3.

Given a Trio of Propositions of Relation, of which every two contain a pair of codivisional Classes, and which are proposed as a Syllogism; to ascertain whether the proposed Conclusion is consequent from the proposed Premisses, and, if so, whether it is complete.

The Rules, for doing this, are as follows :—

- (1) Name the ' Universe of Discourse '.
- (2) Construct a Dictionary, making m and m' (or m and m') represent the pair of codivisional Classes contained in the first two Propositions, and x and y the other two Classes contained in them.
- (3) Translate the given Propositions into abstract form, the first in terms of x and m , the second in terms of y and m , and the third in terms of x and y .
- (4) Take the proposed Premisses, and ascertain, by the process described in § 2, what Conclusion, if any, is consequent from them.
- (5) If there be *no* Conclusion, say so.
- (6) If there *be* a Conclusion, compare it with the proposed Conclusion, and pronounce accordingly.

I will now work out in their briefest form, as models for the Reader to imitate in working examples, six Problems.

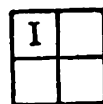
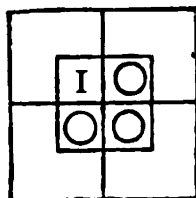
(1).

" All soldiers are strong ;
All soldiers are brave.

Some strong men are brave."

Univ. " *men* " ; m = soldiers ; x = strong ; y = brave.

“All m are x ;
All m are y .”



\therefore “Some x are y ”

Hence proposed Conclusion is right.

(2)

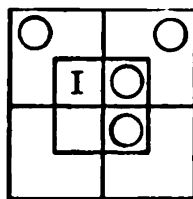
“I admire these pictures ;

When I admire anything I wish to examine it thoroughly.

I wish to examine some of these pictures thoroughly.”

Univ. “things” ; m = admired by me ; x = these pictures ;
 y = which I wish to examine thoroughly.

“All x are m ;
All m are y .”



\therefore “All x are y ”.

Hence proposed Conclusion, “Some x are y ”, is *incomplete*,
the *complete* one being “I wish to examine *all* these pictures
thoroughly”.

(3)

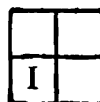
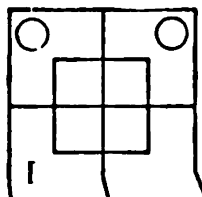
“None but the brave deserve the fair ;

Some braggarts are cowards.

Some braggarts do not deserve the fair.”

Univ. “persons” ; m = brave ; x = deserving of the fair ; y =
braggarts.

“No m' are x ;
Some y are m' .”



\therefore “Some y are x' ”

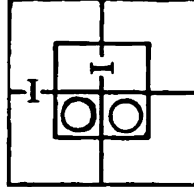
Hence proposed Conclusion is right.

(4)

“ All soldiers can march ;
 Some babies are not soldiers.
 Some babies cannot march ”.

Univ. “ persons ” ; m = soldiers ; x = able to march ; y = babies.

“ All m are x ;
 Some y are m' .”



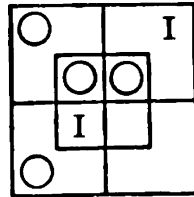
There is no
 Conclusion.

(5)

“ All selfish men are unpopular ;
 All obliging men are popular.
 All obliging men are unselfish ”.

Univ. “ men ” ; m = popular ; x = selfish ; y = obliging.

“ All x are m' ;
 All y are m ”.



∴ “ All x are y' ;
 All y are x' .”

Hence proposed Conclusion, “ All y are x' ,” is *incomplete*, the *complete* one containing, in addition, “ All selfish men are disobliging ”.

(6)

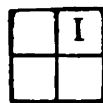
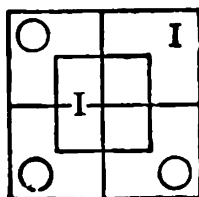
“ Any one, who wishes to go by the train and cannot get a conveyance, and has not enough time to walk to the station, must run ;

This party of tourists wish to go by the train and cannot get a conveyance, but they have plenty of time to walk to the station.

This party of tourists need not run.”

Univ. “ persons wishing to go by &c., and unable to get &c.” ;
 m = *having* enough time to walk &c. ; x = *needing* to run ;
 y = *these* tourists.

"All m' are x ;
All y are m ."



\therefore Some x are y' ."

Hence proposed Conclusion, "All y are x' ," is *wrong*, the *right* one being "Some persons, who wish to go by the train and cannot get a conveyance, and who must run, are not members of this party of tourists."

[Here is *another* opportunity, gentle Reader, for playing a trick on your innocent friend. Put the proposed Syllogism before him, and ask him what he thinks of the Conclusion.

He will reply "Why, it's perfectly correct, of course! And if your precious Logic-book tells you it *isn't*, don't believe it! You don't mean to tell me those tourists *need* to run? If I were one of them, and knew the *Premisses* to be true, I should be *quite* clear that I *needn't* run—and I *should walk*!"

And *you* will reply "But suppose there was a mad bull behind you?"

And then your innocent friend will say "Hum! Ha! I must think that over a bit!"]

[Review Tables V—VIII (pp. 46—49). Work Examples
§ 4, 7—12 (p. 100); § 5, 7—12 (p. 101); § 6, 1—10 (p. 106);
§ 7, 1—6 (p. 107).]

BOOK VI.

THE METHOD OF SUBSCRIPTS.

CHAPTER I.

INTRODUCTORY.

LET us agree that " x_1 " shall mean "Some x exist", that " xy_1 " shall mean "Some xy exist", and so on. Such a Proposition may be called an '**Entity**'.

[Note that, when there are *two* letters in the expression, it does not in the least matter which stands *first*: " xy_1 " and " yx_1 " mean exactly the same.]

Also that " x_0 " shall mean "No x exist", that " xy_0 " shall mean "No xy exist", and so on. Such a Proposition may be called a '**Nullity**'.

Also that " \dagger " shall mean "and".

[Thus " $ab_1 \dagger cd_0$ " means "Some ab exist and no cd exist".]

Also that " \mathbb{P} " shall mean "would, if true, prove".

[Thus, " $x_0 \mathbb{P} xy_0$ " means "The Proposition 'No x exist' would, if true, prove the Proposition 'No xy exist'".]

When two Letters are both of them accented, or both *not* accented, they are said to have '**Like Signs**', or to be '**Like**': when one is accented, and the other not, they are said to have '**Unlike Signs**' or to be '**Unlike**'.

CHAPTER II.

REPRESENTATION OF PROPOSITIONS OF RELATION.

LET us take, first, the Proposition "Some x are y ".

This, we know, is equivalent to the Proposition of Existence "Some xy exist". Hence it may be represented by the expression " xy_1 ".

The Converse Proposition "Some y are x " may of course be represented by the *same* expression, viz. " xy_1 ".

Similarly we may represent the three similar Pairs of Converse Propositions, viz.—

"Some x are y' " = "Some y' are x ",

"Some x' are y " = "Some y are x' ",

"Some x' are y' " = "Some y' are x' ".

Let us take, next, the Proposition "No x are y ".

This, we know, is equivalent to the Proposition of Existence "No xy exist". Hence it may be represented by the expression " xy_0 ".

The Converse Proposition "No y are x " may of course be represented by the *same* expression, viz. " xy_0 ".

Similarly we may represent the other three Pairs of Converse Propositions, viz.

"No x are y' " = "No y' are x ",

"No x' are y " = "No y are x' ",

"No x' are y' " = "No y' are x' ".

Let us take, next, the Proposition "All x are y ".

Now it is evident that the Double Proposition of Existence "Some x exist and no xy' exist" tells us that *some* x -Things exist, but that *none* of them have the Attribute y' : that is, it tells us that *all* of them have the Attribute y : that is, it tells us that "All x are y ".

Also it is evident that the expression " $x_1 \dagger xy'_0$ " represents this Double Proposition.

Hence it also represents the Proposition "All x are y ".

[The Reader will perhaps be puzzled by the statement that the Proposition "All x are y " is equivalent to the Double Proposition "Some x exist and no xy' exist," remembering that it was stated, at p. 33, to be equivalent to the Double Proposition "Some x are y and no x are y' " (i.e. "Some xy exist and no xy' exist"). The explanation is that the Proposition "Some xy exist" contains *superfluous information*. "Some x exist" is enough for our purpose.]

This expression may be written in a shorter form, viz. " $x_1y'_0$ ", since *each* Subscript takes effect back to the *beginning* of the expression.

Similarly we may represent the seven similar Propositions "All x are y' ", "All x' are y ", "All x' are y' ", "All y are x ", "All y are x' ", "All y' are x ", and "All y' are x' ".

[The Reader should make out all these for himself.]

It will be convenient to remember that, in translating a Proposition from abstract form into subscript form, or *vice versa*, the Predicate *changes sign* (that is, changes from positive to negative, or else from negative to positive).

[Thus, the Proposition "All y are x' " becomes " y_1x_0 ", where the Predicate changes from x' to x .

Again, the expression " $x'_1y'_0$ " becomes "All x' are y ", where the Predicate changes from y' to y .]

CHAPTER III.

SYLLOGISMS.

§ 1.

Representation of Syllogisms.

WE already know how to represent each of the three Propositions of a Syllogism in subscript form. When that is done, all we need, besides, is to write the three expressions in a row, with “†” between the Premisses, and “ℙ” before the Conclusion.

[Thus the Syllogism

“No x are m' ;
All m are y .
∴ No x are y' .”

may be represented thus:—

$xm'_0 \dagger m_1y'_0 \mathbb{P} xy'_0$

When a Proposition has to be translated from concrete form into subscript form, the Reader will find it convenient, just at first, to translate it into *abstract* form, and *thence* into subscript form. But, after a little practice, he will find it quite easy to go straight from concrete form to subscript form.]

§ 2.

Formulae for solving Problems in Syllogisms.

When once we have found, by Diagrams, the Conclusion to a given Pair of Premisses, and have represented the Syllogism in subscript form, we have a *Formula*, by which we can at once find, without having to use Diagrams again, the Conclusion to any *other* Pair of Premisses having the *same* subscript forms.

[Thus, the expression

$$xm_0 \dagger ym'_0 \P xy_0$$

is a *Formula*, by which we can find the Conclusion to any Pair of Premisses whose subscript forms are

$$xm_0 \dagger ym'_0$$

For example, suppose we had the Pair of Propositions

“No gluttons are healthy ;
No unhealthy men are strong”.

proposed as Premisses. Taking “men” as our ‘Universe,’ and making m = healthy ; x = gluttons ; y = strong ; we might translate the Pair into abstract form, thus :—

“No x are m ;
No m' are y .”

These, in subscript form, would be

$$xm_0 \dagger m'y_0$$

which are identical with those in our *Formula*. Hence we at once know the Conclusion to be

$$xy_0$$

that is, in abstract form,

“No x are y ” ;

that is, in concrete form,

“No gluttons are strong”.]

I shall now take three different forms of Pairs of Premisses, and work out their Conclusions, once for all, by *Diagrams* ; and thus obtain some useful *Formulae*. I shall call them “Fig. I”, “Fig. II”, and “Fig. III”.

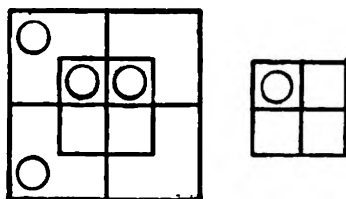
Fig. I.

This includes any Pair of Premisses which are both of them Nullities, and in which the Eliminands have Unlike Signs.

The simplest case is

$$xm_0 \dagger ym'_0$$

These, worked on Diagrams, yield the Conclusion xy_0



In this case we see that the Conclusion is a Nullity, and that the Retinends have kept their Signs.

And we should find this Rule to hold good with *any* Pair of Premisses which fulfil the given conditions.

[The Reader had better satisfy himself of this, by working out, on Diagrams, several varieties, such as

$$\begin{aligned} m_1x_0 \dagger ym'_0 & \text{ (which } \mathbb{P} xy_0) \\ xm'_0 \dagger m_1y_0 & \text{ (which } \mathbb{P} xy_0) \\ x'm_0 \mathbb{P} ym'_0 & \text{ (which } \mathbb{P} x'y_0) \\ m'_1x'_0 \dagger m_1y'_0 & \text{ (which } \mathbb{P} x'y'_0). \end{aligned}$$

If either Retinend is asserted in the *Premisses* to exist, of course it may be so asserted in the *Conclusion*.

Hence we get two *Variants* of Fig. I, viz.

- (a) where *one* Retinend is so asserted ;
- (β) where *both* are so asserted.

[The Reader had better work out, on Diagrams, examples of these two Variants, such as

$$\begin{aligned} m_1x_0 \dagger y_1m'_0 & \text{ (which proves } y_1x_0) \\ x_1m'_0 \dagger m_1y_0 & \text{ (which proves } x_1y_0) \\ x'_1m_0 \dagger y_1m'_0 & \text{ (which proves } x'_1y_0 \dagger y_1x'_0). \end{aligned}$$

The Formula, to be remembered, is

$$xm_0 \dagger ym'_0 \mathbb{P} xy_0$$

with the following two Rules:—

- (1) *Two Nullities, with Unlike Eliminands, yield a Nullity, in which both Retinends keep their Signs.*

- (2) *A Retinend, asserted in the Premisses to exist, may be so asserted in the Conclusion.*

[Note that Rule (1) is merely the Formula expressed in words.]

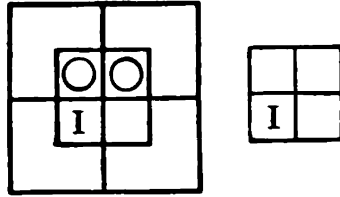
Fig. II.

This includes any Pair of Premisses, of which one is a Nullity and the other an Entity, and in which the Eliminands have Like Signs.

The simplest case is

$$xm_0 \dagger ym_1$$

These, worked on Diagrams, yield the Conclusion $x'y_1$



In this case we see that the Conclusion is an Entity, and that the Nullity-Retinend changes its Sign.

And we should find this Rule to hold good with *any* Pair of Premisses which fulfil the given conditions.

[The Reader had better satisfy himself of this, by working out, on Diagrams, several varieties, such as

$$\begin{aligned} &x'm_0 \dagger ym_1 \text{ (which } \mathbb{P} xy_1) \\ &x_1m'_0 \dagger y'm'_1 \text{ (which } \mathbb{P} x'y'_1) \\ &m_1x_0 \dagger y'm_1 \text{ (which } \mathbb{P} x'y'_1). \end{aligned}$$

The Formula, to be remembered, is,

$$xm_0 \dagger ym_1 \mathbb{P} x'y_1$$

with the following Rule:—

A Nullity and an Entity, with Like Eliminands, yield an Entity, in which the Nullity-Retinend changes its Sign.

[Note that this Rule is merely the Formula expressed in words.]

Fig. III.

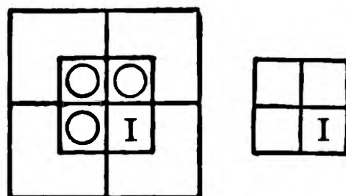
This includes any Pair of Premisses which are both of them Nullities, and in which the Eliminands have Like Signs and are asserted to exist.

The simplest case is

$$xm_0 \dagger ym_0 \dagger m_1$$

[Note that " m_1 " is here stated *separately*, because it does not matter in *which* of the two Premisses it occurs: so that this includes the *three* forms " $m_1x_0 \dagger ym_0$ ", " $xm_0 \dagger m_1y_0$ ", and " $m_1x_0 \dagger m_1y_0$ ".]

These, worked on Diagrams, yield the Conclusion $x'y'_1$



In this case we see that the Conclusion is an Entity, and that *both* Retinends have changed their Signs.

And we should find this Rule to hold good with *any* Pair of Premisses which fulfil the given conditions.

[The Reader had better satisfy himself of this, by working out, on Diagrams, several varieties, such as

$$\begin{aligned} &x'm_0 \dagger m_1y_0 \text{ (which } \mathbb{P} \ x'y'_1) \\ &m'_1x_0 \dagger m'y'_0 \text{ (which } \mathbb{P} \ x'y'_1) \\ &m_1x'_0 \dagger m_1y'_0 \text{ (which } \mathbb{P} \ xy_1). \end{aligned}$$

The Formula, to be remembered, is

$$xm_0 \dagger ym_0 \dagger m_1 \mathbb{P} \ x'y'_1$$

with the following Rule (which is merely the Formula expressed in words):—

Two Nullities, with Like Eliminands asserted to exist, yield an Entity, in which both Retinends change their Signs.

In order to help the Reader to remember the peculiarities and Formulæ of these three Figures, I will put them all together in one Table.

TABLE IX.

Fig. I.

$$xm_0 \dagger ym'_0 \P xy_0$$

Two Nullities, with Unlike Eliminands, yield a Nullity, in which both Retinends keep their Signs.

A Retinend, asserted in the Premisses to exist, may be so asserted in the Conclusion.

Fig. II.

$$xm_0 \dagger ym_1 \P x'y_1$$

A Nullity and an Entity, with Like Eliminands, yield an Entity, in which the Nullity-Retinend changes its Sign.

Fig. III.

$$xm_0 \dagger ym_0 \dagger m_1 \P x'y'_1$$

Two Nullities, with Like Eliminands asserted to exist, yield an Entity, in which both Retinends change their Signs.

I will now work out, by these Formulæ, as models for the Reader to imitate, all the Problems in Syllogisms which have been already worked, by Diagrams, in Book V., Chap. II.

(1) [see p. 64]

“No son of mine is dishonest ;

People always treat an honest man with respect.”

Univ. “men” ; m = honest ; x = my sons ; y = treated with respect.

$$xm'_0 \dagger m_1y'_0 \P xy'_0 \text{ [Fig. I.]}$$

i.e. “No son of mine ever fails to be treated with respect.”

(2) [see p. 64]

“All cats understand French ;
Some chickens are cats.”

Univ. “creatures” ; m = cats ; x = understanding French ;
 y = chickens

$$m_1x'_0 \dagger ym_1 \mathbb{P} xy_1 \text{ [Fig II.}$$

i.e. “Some chickens understand French.”

(3) [see p. 64]

“All diligent students are successful ;
All ignorant students are unsuccessful.”

Univ. “students” ; m = successful ; x = diligent ; y = ignorant.

$$x_1m'_0 \dagger y_1m_0 \mathbb{P} x_1y_0 \dagger y_1x_0 \text{ [Fig. I } (\beta).$$

i.e. “All diligent students are learned ; and all ignorant students are idle.”

(4) [see p. 65]

“Of the prisoners who were put on their trial at the last
Assizes, all, against whom the verdict ‘guilty’ was
returned, were sentenced to imprisonment ;

Some, who were sentenced to imprisonment, were also
sentenced to hard labour.”

Univ. “prisoners who were put on &c” ; m = sentenced to
imprisonment ; x = against whom the verdict ‘guilty’ was
returned ; y = sentenced to hard labour.”

$$x_1m'_0 \dagger my_1 \mathbb{P} \text{ nothing.}$$

Hence there is no Conclusion.

(5) [see p. 66]

“All soldiers are strong ;
All soldiers are brave.

Some strong men are brave.”

Univ. “men” ; m = soldiers ; x = strong ; y = brave.

$$m_1x'_0 \dagger m_1y'_0 \mathbb{P} xy_1 \text{ [Fig. III.}$$

Hence proposed Conclusion is right.

(6) [see p. 67]

“I admire these pictures ;

When I admire anything, I wish to examine it thoroughly.

I wish to examine some of these pictures thoroughly.”

Univ. “pictures” ; m = admired by me ; x = these ; y = which I wish to examine thoroughly.

$$x_1 m'_0 \dagger m_1 y'_0 \mathbb{P} x_1 y'_0 \text{ [Fig. I (a).]}$$

Hence proposed Conclusion, xy_1 , is *incomplete*, the *complete* one being “I wish to examine *all* these pictures thoroughly.”

(7) [see p. 67]

“None but the brave deserve the fair ;

Some braggarts are cowards.

Some braggarts do not deserve the fair.”

Univ. “persons” ; m = brave ; x = deserving of the fair y = braggarts.

$$m'x_0 \dagger ym'_1 \mathbb{P} x'y_1 \text{ [Fig. II.]}$$

Hence proposed Conclusion is right.

(8) [see p. 68]

“All soldiers can march ;

Some babies are not soldiers.

Some babies cannot march.”

Univ. “persons” ; m = soldiers ; x = can march ; y = babies.

$$m_1 x'_0 \dagger ym'_1 \mathbb{P} \text{ nothing.}$$

Hence there is no Conclusion.

(9) [see p. 68]

“All selfish men are unpopular ;

All obliging men are popular.

All obliging men are unselfish.”

Univ. “men” ; m = popular ; x = selfish ; y = obliging.

$$x_1 m_0 \dagger y_1 m'_0 \mathbb{P} (x_1 y \dagger y_1 x_0) \text{ [Fig. I (\beta).]}$$

Hence proposed Conclusion, $y_1 x_0$, is *incomplete*, the *complete* one containing, in addition, “All selfish men are disobliging.”

(10) [see p. 69]

“ Any one, who wishes to go by the train and cannot get a conveyance, and has not enough time to walk to the station, must run ;

This party of tourists wish to go by the train and cannot get a conveyance, but they have plenty of time to walk to the station.

This party of tourists need not run.”

Univ. “ persons wishing to go by &c. and unable to get &c.” ;
 m = having enough time to walk &c. ; x = needing to run ;
 y = these tourists.

$m'_1x'_0 \vdash y_1m'_0 \text{ } \text{P} \text{ } xy'_1$ [Fig. III.

Hence proposed Conclusion, $y_1x'_0$, is *wrong*, the *right* one being “ Some persons, who wish to go by the train and cannot get a conveyance, and who must run, are not members of this party of tourists.”

[Work Examples § 4, 1—10 (p. 100); § 5, 13—24 (p. 101); § 6, 1—6 (p. 106); § 7, 1—6 (p. 107).]

§ 3.

Fallacies.

Any argument which *deceives* us, by seeming to prove what it does not really prove, may be called a ‘**Fallacy**’ (derived from the Latin verb *fallo* “I deceive”): but the particular kind, to be now discussed, consists of a Pair of Propositions, which are proposed as the Premisses of a Syllogism, but yield no Conclusion.

When each of the proposed Premisses is a Proposition in *I*, or *E*, or *A*, (the only kinds with which we are now concerned,) the Fallacy may be detected by the ‘Method of Diagrams,’ by simply setting them out on a Trilateral

Diagram, and observing that they yield no information which can be transferred to the Biliteral Diagram.

But suppose we were working by the 'Method of *Subscripts*, and had to deal with a Pair of proposed Premisses, which happened to be a 'Fallacy,' how could we be certain that they would not yield any Conclusion?

Our best plan is, I think, to deal with *Fallacies* in the same way as we have already dealt with *Syllogisms*: that is, to take certain forms of Pairs of Propositions, and to work them out, once for all, on the Triliteral Diagram, and ascertain that they yield *no* Conclusion; and then to record them, for future use, as *Formulae for Fallacies*, just as we have already recorded our three *Formulae for Syllogisms*.

Now, if we were to record the two Sets of Formulae in the *same* shape, viz. by the Method of Subscripts, there would be considerable risk of confusing the two kinds. Hence, in order to keep them distinct, I propose to record the Formulae for *Fallacies* in *words*, and to call them "Forms" instead of "Formulae."

Let us now proceed to find, by the Method of Diagrams, three "Forms of Fallacies," which we will then put on record for future use. They are as follows:—

- (1) Fallacy of Like Eliminands not asserted to exist.
- (2) Fallacy of Unlike Eliminands with an Entity-Premiss.
- (3) Fallacy of two Entity-Premisses.

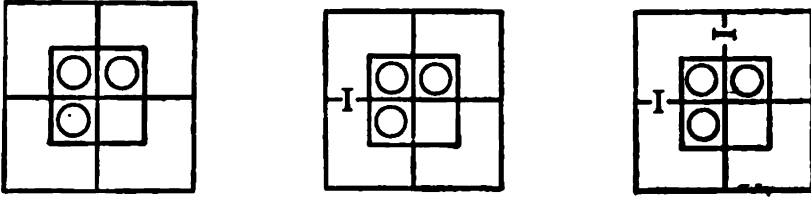
These shall be discussed separately.

(1) *Fallacy of Like Eliminands not asserted to exist.*

It is evident that neither of the given Propositions can be an *Entity*, since that kind asserts the *existence* of both of its Terms (see p. 20). Hence they must both be *Nullities*.

Hence the given Pair may be represented by $(xm_0 \dagger ym_c)$, with or without x_1, y_1 .

These, set out on Trilateral Diagrams, are

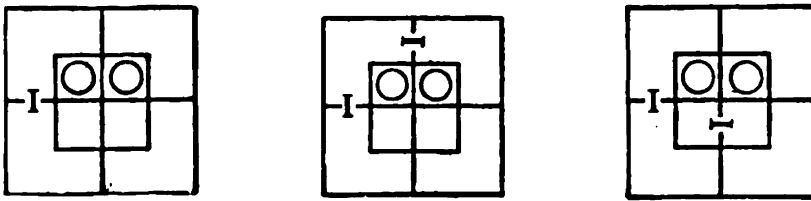


each of which, it is evident, fails to yield a Conclusion.

(2) *Fallacy of Unlike Eliminands with an Entity-Premiss.*

Here the given Pair may be represented by $(xm_0 \dagger ym'_1)$ with or without x_1 or m_1 .

These, set out on Trilateral Diagrams, are

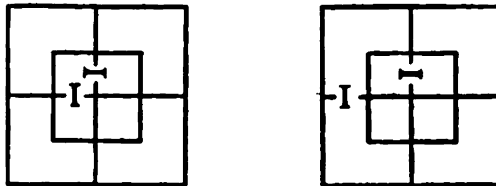


each of which, it is evident, fails to yield a Conclusion.

(3) *Fallacy of two Entity-Premisses.*

Here the given Pair may be represented by either $(xm_1 \dagger ym_1)$ or $(xm_1 \dagger ym'_1)$.

These, set out on Trilateral Diagrams, are



each of which, it is evident, fails to yield a Conclusion.

§ 4.

Method of proceeding with a given Pair of Propositions.

Let us suppose that we have before us a Pair of Propositions of Relation, which contain between them a pair of codivisional Classes, and that we wish to ascertain what Conclusion, if any, is consequent from them. We translate them, if necessary, into subscript-form, and then proceed as follows :—

(1) We examine their Subscripts, in order to see whether they are

- (a) a Pair of Nullities ;
- or (b) a Nullity and an Entity ;
- or (c) a Pair of Entities.

(2) If they are a Pair of Nullities, we examine their Eliminands, in order to see whether they are Unlike or Like.

If their Eliminands are *Unlike*, it is a case of Fig. I. We then examine their Retinends, to see whether one or both of them are asserted to *exist*. If *one* Retinend is so asserted, it is a case of Fig. I (a) ; if *both*, it is a case of Fig. I (β).

If their Eliminands are *Like*, we examine them, in order to see whether either of them is asserted to exist. If so, it is a case of Fig. III. ; if not, it is a case of “ Fallacy of Like Eliminands not asserted to exist.”

(3) If they are a Nullity and an Entity, we examine their Eliminands, in order to see whether they are Like or Unlike.

If their Eliminands are *Like*, it is a case of Fig. II. ; if *Unlike*, it is a case of “ Fallacy of Unlike Eliminands with an Entity-Premiss.”

(4) If they are a Pair of Entities, it is a case of “ Fallacy of two Entity-Premisses.”

[Work Examples § 4, 11—20 (p. 100) ; § 5, 1—12 (p. 101) ; § 6, 7—12 (p. 106) ; § 7, 7—12 (p. 107).]

BOOK VII.

SORITES.

CHAPTER I.

INTRODUCTORY.

WHEN a Set of three or more Biliteral Propositions are so related that two of them, taken together, yield a Conclusion, which, taken with another of them, yields another Conclusion, and so on, until all have been taken, it is evident that, if the original Set were true, the last Conclusion would *also* be true.

Such a Set, with the last Conclusion tacked on, is called a '**Sorites**'; the original Set of Propositions is called its '**Premisses**'; each of the intermediate Conclusions is called a '**Partial Conclusion**' of the Sorites; the last Conclusion is called its '**Complete Conclusion**,' or, more briefly, its '**Conclusion**'; the Terms, used as Eliminands in the Syllogisms, are called its '**Eliminands**'; and the other two Terms are called its '**Retinends**'.

[Note that each *Partial Conclusion* contains one or two *Eliminands*; but that the *Complete Conclusion* contains *Retinends* only.]

The Conclusion is said to be '**consequent**' from the Premisses; for which reason it is usual to prefix to it the word "Therefore" (or the symbol " \therefore ").

[Note that the question, whether the Conclusion is or is not *consequent* from the Premisses, is not affected by the *actual* truth or falsity of any one of the Propositions which make up the Sorites, but depends entirely on their *mutual relation*.

As a specimen-Sorites, let us take the following Set of 5 Propositions :—

- (1) "No a are b' ;
- (2) All b are c ;
- (3) All c are d ;
- (4) No e' are a' ;
- (5) All h are e' ".

Here the first and second, taken together, yield "No a are c' "

This, taken along with the third, yields "No a are d' ".

This, taken along with the fourth, yields "No d' are e' ".

And this, taken along with the fifth, yields "All h are d ".

Hence, if the original Set were true, this would *also* be true.

Hence the original Set, with this tacked on, is a *Sorites*; the original Set is its *Premisses*; the Proposition "All h are d " is its *Conclusion*; the Terms a, b, c, e are its *Eliminands*; and the Terms d and h are its *Retinends*.

Hence we may write the whole Sorites thus :—

"No a are b' ;
 All b are c ;
 All c are d ;
 No e' are a' ;
 All h are e' ;
 \therefore All h are d "

In the above Sorites, the 3 Partial Conclusions are the Propositions "No a are c' ", "No a are d' ", "No d' are e' "; but, if the Premisses were arranged in other ways, other Partial Conclusions would be obtained. Thus, the order 41523 yields the Partial Conclusions "No e' are b' ", "All h are b ", "All h are c ". There are altogether *nine* Partial Conclusions to this Sorites, which the Reader will find it an interesting task to make out for himself.]

CHAPTER II.

PROBLEMS IN SORITES.

§ 1.

Introductory.

THE Problems we shall have to solve are of the following form :—

“Given three or more Propositions of Relation, which are proposed as Premisses : to ascertain what Conclusion, if any, is consequent from them.”

We will limit ourselves, at present, to Problems which can be worked by the Formulæ of Fig. I. (See p. 75.) Those, that require *other* Formulæ, are rather too hard for beginners.

Such Problems may be solved by either of two Methods, viz.

- (1) The Method of Separate Syllogisms ;
- (2) The Method of Underscoring.

These shall be discussed separately.

§ 2.

Solution by Method of Separate Syllogisms.

The Rules, for doing this, are as follows:—

- (1) Name the ' Universe of Discourse '.
- (2) Construct a Dictionary, making *a*, *b*, *c*, &c. represent the Terms.
- (3) Put the Proposed Premisses into subscript form.
- (4) Select two which, containing between them a pair of codivisional Classes, can be used as the Premisses of a Syllogism.
- (5) Find their Conclusion by Formula.
- (6) Find a third Premiss which, along with this Conclusion, can be used as the Premisses of a second Syllogism.
- (7) Find a second Conclusion by Formula.
- (8) Proceed thus, until all the proposed Premisses have been used.
- (9) Put the last Conclusion, which is the Complete Conclusion of the Sorites, into concrete form.

[As an example of this process, let us take, as the proposed Set of Premisses,

- (1) " All the policemen on this beat sup with our cook ;
- (2) No man with long hair can fail to be a poet ;
- (3) ✓ Amos Judd has never been in prison ;
- (4) Our cook's ' cousins ' all love cold mutton ;
- (5) None but policemen on this beat are poets ;
- (6) None but her ' cousins ' ever sup with our cook ;
- (7) Men with short hair have all been in prison."

Let Univ. be " men " ; *a* = Amos Judd ; *b* = cousins of &c. ;
c = having been in prison ; *d* = long-haired ;
e = loving cold mutton ; *h* = poets ; *k* = policemen on &c. ;
l = supping with our cook.

We now have to put the proposed Premisses into *subscript* form. Let us begin by putting them into *abstract* form. The result is

- (1) "All k are l ;
- (2) No d are h ';
- (3) All a are c ';
- (4) All b are e ;
- (5) No k' are h ;
- (6) No b' are l ';
- (7) All d' are c '".

And it is now easy to put them into *subscript* form, as follows:—

- (1) $k_1 l'_0$
- (2) dh'_0
- (3) $a_1 c_0$
- (4) $b_1 e'_0$
- (5) $k'_1 h_0$
- (6) $b'l'_0$
- (7) $d'_1 c'_0$

We now have to find a pair of Premisses which will yield a Conclusion. Let us begin with No. (1), and look down the list, till we come to one which we can take along with it, so as to form Premisses belonging to Fig. I. We find that No. (5) will do, since we can take k as our Eliminand. So our first syllogism is

- (1) $k_1 l'_0$
- (5) $k'_1 h_0$
- $\therefore l'h_0 \dots (8)$

We must now begin again with $l'h_0$, and find a Premiss to go along with it. We find that No. (2) will do, h being our Eliminand. So our next Syllogism is

- (8) $l'h_0$
- (2) dh'_0
- $\therefore l'd_0 \dots (9)$

We have now used up Nos. (1), (5), and (2), and must search among the others for a partner for $l'd_0$. We find that No. (6) will do. So we write

- (9) $l'd_0$
- (6) $b'l'_0$
- $\therefore db'_0 \dots (10)$

N w what can we take with db'_0 ? No. (4) will do.

- (10) db'_0
- (4) $b_1 e'_0$
- $\therefore de'_0 \dots (11)$

Along with *this* we may take No. (7).

$$\begin{array}{l} (11) \ d'e'_0 \\ (7) \ d'_1c'_0 \\ \therefore e'e'_0 \dots (12) \end{array}$$

And along with *this* we may take No. (3).

$$\begin{array}{l} (12) \ e'e'_0 \\ (3) \ a_1c_0 \\ \therefore a_1e'_0 \end{array}$$

This, translated into *abstract* form, is

“All *a* are *e*”;

and this, translated into *concrete* form, is

“Amos Judd loves cold mutton.”]

[In actually *working* this Problem, the above explanations would, of course, be omitted, and all, that would appear on paper, would be as follows :—

$$\begin{array}{l} (1) \ k_1l'_0 \\ (2) \ dh'_0 \\ (3) \ a_1c_0 \\ (4) \ b_1e'_0 \\ (5) \ k'h_0 \\ (6) \ b'l_0 \\ (7) \ d'_1c'_0 \end{array}$$

$$\begin{array}{lll} (1) \ k_1l'_0 & (8) \ l'h_0 & (9) \ l'd_0 \\ (5) \ k'h_0 & (2) \ dh'_0 & (6) \ b'l_0 \\ \therefore l'h_0 \dots (8) & \therefore l'd_0 \dots (9) & \therefore db_0 \dots (10) \end{array}$$

$$\begin{array}{lll} (10) \ db'_0 & (11) \ de'_0 & (12) \ e'e'_0 \\ (4) \ b_1e'_0 & (7) \ d'_1c'_0 & (3) \ a_1c_0 \\ \therefore de'_0 \dots (11) & \therefore e'e'_0 \dots (12) & \therefore a_1e'_0] \end{array}$$

§ 3.

Solution by Method of Underscoring.

Consider the Syllogism

$$xm_0 \dagger ym'_0 \mathbb{P} xy_0$$

We see that, in order to get the Conclusion, we eliminate the m and the m' , and write the x and the y together in one expression.

Now, if we agree to mark the m and the m' as eliminated, and to read the two expressions together, as if they were written in one, the two Premisses will then exactly represent the *Conclusion*, and we need not write it out separately.

Let us agree to mark the eliminated letters by *underscoring* them, putting a *single* score under the *first*, and a *double* one under the *second*.

The two Premisses would now be

$$x\underline{m}_0 \dagger y\underline{m}'_0$$

which we should read as " xy_0 ".

In re-arranging the Series of Premisses, it will be convenient to *omit all subscripts*. As to the "0's" we may always *suppose* them written, and, as to the "1"s, we are not concerned to know *which* Terms are asserted to *exist*, except those which appear in the *Complete Conclusion*; and for *them* it will be easy enough to refer to the original list.

[I will now go through the process of solving, by this method, the example worked in § 2.

The Data are

$$\overset{1}{k_1 l'_0} \dagger \overset{2}{d h'_0} \dagger \overset{3}{a_1 c_0} \dagger \overset{4}{b_1 e'_0} \dagger \overset{5}{k' h_0} \dagger \overset{6}{b' l_0} \dagger \overset{7}{d'_1 c'_0}$$

We begin by writing down the first, omitting the subscripts, viz. :—

$$\overset{1}{k l'}$$

We have now to find a Premiss which can be combined with this, *i.e.*, a Premiss containing either k' or l . The first we find is No. 5 ; and, tacking this on, we get

$$\overset{1}{k l'} \dagger \overset{5}{k' h}$$

To get the *Conclusion* from these, k and k' must be eliminated, and what remains must be taken as one expression. So we *underscore* them, and the result is

$$\overset{1}{\underline{k l'}} \dagger \overset{5}{\underline{k' h}}, \text{ which we read as } l' h.$$

We must now find a Premiss containing either l or h' . Looking along the row, we at once fix on No. 2, and tack it on, thus getting

$$\overset{1}{\underline{k l'}} \dagger \overset{5}{\underline{k' h}} \dagger \overset{2}{d h'}$$

Now these are really equivalent to $(l' h \dagger d h')$, in which h and h' must be eliminated, and what remains taken as one expression. So we *underscore* them, and thus get

$$\overset{1}{\underline{k l'}} \dagger \overset{5}{\underline{k' h}} \dagger \overset{2}{d \underline{h'}}, \text{ which we read as } l' d.$$

We now want a Premiss containing l or d' . No. 6 will do. Tacking it on, we get

$$\overset{1}{\underline{k l'}} \dagger \overset{5}{\underline{k' h}} \dagger \overset{2}{d \underline{h'}} \dagger \overset{6}{b' l}$$

These are really equivalent to $(l' d \dagger b' l)$. So we *underscore* l' and l , and thus get

$$\overset{1}{\underline{k l'}} \dagger \overset{5}{\underline{k' h}} \dagger \overset{2}{d \underline{h'}} \dagger \overset{6}{\underline{b' l}}, \text{ which we read as } d b'.$$

We now want a Premiss containing d' or b . No. 4 will do. We tack it on, and get

$$\overset{1}{\underline{k l'}} \dagger \overset{5}{\underline{k' h}} \dagger \overset{2}{d \underline{h'}} \dagger \overset{6}{\underline{b' l}} \dagger \overset{4}{b c'}$$

Here we *underscore* b' and b , and thus get

$$\overset{1}{\underline{k l'}} \dagger \overset{5}{\underline{k' h}} \dagger \overset{2}{d \underline{h'}} \dagger \overset{6}{\underline{b' l}} \dagger \overset{4}{\underline{b c'}}, \text{ which reads as } d c'.$$

We now want a Premiss containing d' or e . No. 7 will do
We now have

$$\overset{1}{k} \overset{5}{l'} \vdash \overset{2}{k'} \overset{6}{h} \vdash \overset{4}{d} \overset{7}{h'} \vdash \overset{4}{b'} \overset{7}{l} \vdash \overset{4}{b} \overset{7}{e'} \vdash \overset{7}{d'} \overset{7}{c'}$$

Here we underscore d and d' , and thus get

$$\overset{1}{k} \overset{5}{l'} \vdash \overset{2}{k'} \overset{6}{h} \vdash \overset{4}{\underline{d}} \overset{7}{h'} \vdash \overset{4}{b'} \overset{7}{l} \vdash \overset{4}{b} \overset{7}{e'} \vdash \overset{7}{\underline{d'}} \overset{7}{c'}, \text{ which reads as } e'c'.$$

We now want a Premiss containing e or c . No. 3 will do—in fact *must* do, as it is the only one left. We thus get

$$\overset{1}{k} \overset{5}{l'} \vdash \overset{2}{k'} \overset{6}{h} \vdash \overset{4}{\underline{d}} \overset{7}{h'} \vdash \overset{4}{b'} \overset{7}{l} \vdash \overset{4}{b} \overset{7}{e'} \vdash \overset{7}{\underline{d'}} \overset{7}{c'} \vdash \overset{3}{a} \overset{3}{c}$$

Here we underscore c' and c ; and, as the whole thing now reads as $e'a$, we may tack on $e'a_0$ as the *Conclusion*, thus:—

$$\overset{1}{k} \overset{5}{l'} \vdash \overset{2}{k'} \overset{6}{h} \vdash \overset{4}{\underline{d}} \overset{7}{h'} \vdash \overset{4}{b'} \overset{7}{l} \vdash \overset{4}{b} \overset{7}{e'} \vdash \overset{7}{\underline{d'}} \overset{7}{c'} \vdash \overset{3}{a} \overset{3}{\underline{c}} \vdash \overset{3}{a} \overset{3}{e'} \vdash \overset{3}{a} \overset{3}{c_0}$$

We now look along the row of Data, to see whether e' or a has been given as *existent*. We find that a has been so given in No. 3. So we add this fact to the Conclusion, which now stands as

$$\vdash e'a_0 \vdash a_1, \text{ i.e. } \vdash a_1e'_0$$

which, in abstract form, is “All a are e .”

In actually *working* this Problem, the above explanations would, of course, be omitted, and all, that would appear on paper, would be as follows:—

$$\overset{1}{k_1} \overset{2}{l'_0} \vdash \overset{2}{d} \overset{3}{h'_0} \vdash \overset{3}{a_1} \overset{4}{c_0} \vdash \overset{4}{b_1} \overset{5}{e'_0} \vdash \overset{5}{k'} \overset{6}{h_0} \vdash \overset{6}{b'} \overset{7}{l'_0} \vdash \overset{7}{d'_1} \overset{7}{c'_0}$$

$$\overset{1}{k} \overset{5}{l'} \vdash \overset{2}{k'} \overset{6}{h} \vdash \overset{4}{\underline{d}} \overset{7}{h'} \vdash \overset{4}{b'} \overset{7}{l} \vdash \overset{4}{b} \overset{7}{e'} \vdash \overset{7}{\underline{d'}} \overset{7}{c'} \vdash \overset{3}{a} \overset{3}{\underline{c}} \vdash \overset{3}{a} \overset{3}{e'_0} \vdash \overset{3}{a_1} \text{ i.e. } \vdash \overset{3}{a_1} \overset{3}{e'_0}$$

i.e. “All a are e .”

If the Reader has carefully gone through this description, he will have seen the work *grow* (as it were) under my hand, and it will be nearly the same thing as actually *watching* me while writing it.

He had better now turn to the Data at the beginning of this Note, and copy them, and cover up the rest of the page, and try to work it out for himself, starting with $k_1l'_0$.

If he fails to bring out the Conclusion $a_1e'_0$, I would advise him to take another piece of paper, and *begin again* !]

I will now work out, in its briefest form, a Sorites of 5 Premises, to serve as a model for the Reader to imitate in working examples.

- (1) "I greatly value everything that John gives me ;
- (2) Nothing but this bone will satisfy my dog ;
- (3) I always take particular care of anything that I greatly value ;
- (4) This bone was a present from John ;
- (5) When I take particular care of anything, I do *not* give it to my dog "

Univ. "things" ; a = given by John to me ; b = given by me to my dog ; c = greatly valued by me ; d = satisfactory to my dog " ; e = taken particular care of by me ; h = this bone.

$$\overset{1}{a_1c'_0} \vdash \overset{2}{h'd_0} \vdash \overset{3}{c_1e'_0} \vdash \overset{4}{h_1a'_0} \vdash \overset{5}{e_1b_0}$$

$$\overset{1}{a\underline{c}'} \vdash \overset{3}{\underline{c}e'} \vdash \overset{4}{h\underline{a}'} \vdash \overset{2}{\underline{h'd}} \vdash \overset{5}{\underline{e}b} \text{ P } db_0$$

i.e. "Nothing, that I give my dog, satisfies him," or, "My dog is not satisfied with *anything* that I give him !"

[Note that, in working a Sorites by this process, we may begin with *any* Premiss we choose. For instance, we might begin with No. 5, and the result would then be

$$\overset{5}{eb} \vdash \overset{3}{\underline{ce}'} \vdash \overset{1}{\underline{ac}'} \vdash \overset{4}{\underline{ha}'} \vdash \overset{2}{\underline{h'd}} \text{ P } bd_0]$$

[Work Examples § 4, 25—30 (p. 100) ; § 5, 25—30 (p. 102) ; § 6, 13—15 (p. 106) ; § 7, 13—15 (p. 108) ; § 8, 1—4, 13, 14, 19, 24 (p. 110) ; § 9, 1—4, 26, 27, 40, 48 (p. 112).]

The Reader, who has successfully grappled with all the Examples hitherto set, and who thirsts, like Alexander the Great, for "more worlds to conquer," may employ his spare energies on the following 17 Examination-Papers. He is recommended not to attempt more than *one* Paper on any one day.

- I. § 4, 31 (p. 100) ; § 5, 31—34 (p. 102) ; § 6, 16, 17 (p. 106) ; § 7, 16 (p. 108) ; § 8, 5, 6 (p. 110) ; § 9, 5, 22, 42 (pp. 112, 115, 118).
- II. § 4, 32 (p. 100) ; § 5, 35—38 (pp. 102, 103) ; § 6, 18 (p. 107) ; § 7, 17, 18 (p. 108) ; § 8, 7, 8 (p. 110) ; § 9, 6, 23, 43 (pp. 112, 115, 118).
- III. § 4, 33 (p. 100) ; § 5, 39—42 (p. 103) ; § 6, 19, 20 (p. 107) ; § 7, 19 (p. 109) ; § 8, 9, 10 (p. 111) ; § 9, 7, 24, 44 (pp. 112, 115, 119).
- IV. § 4, 34 (p. 100) ; § 5, 43—46 (p. 103) ; § 6, 21 (p. 107) ; § 7, 20, 21 (p. 109) ; § 8, 11, 12 (p. 111) ; § 9, 8, 25, 45 (pp. 113, 115, 119).
- V. § 4, 35 (p. 100) ; § 5, 47—50 (p. 103) ; § 6, 22, 23 (p. 107) ; § 7, 22 (p. 109) ; § 8, 15, 16 (p. 111) ; § 9, 9, 28, 46 (pp. 113, 116, 119).
- VI. § 4, 36 (p. 100) ; § 5, 51—54 (p. 103) ; § 6, 24 (p. 107) ; § 7, 23, 24 (p. 109) ; § 8, 17 (p. 111) ; § 9, 10, 29, 47 (pp. 113, 116, 119).
- VII. § 4, 37 (p. 100) ; § 5, 55—58 (pp. 103, 104) ; § 6, 25, 26 (p. 107) ; § 7, 25 (p. 109) ; § 8, 18 (p. 111) ; § 9, 11, 30, 49 (pp. 113, 116, 120).

- VIII. § 4, 38 (p. 100); § 5, 59—62 (p. 104); § 6, 27 (p. 107)
 § 7, 26, 27 (p. 109); § 8, 20 (p. 111); § 9, 12, 31, 5
 (pp. 113, 116, 120).
- IX. § 4, 39 (p. 100); § 5, 63—66 (p. 104); § 6, 28, 29 (p. 107)
 § 7, 28 (p. 109); § 8, 21 (p. 111); § 9, 13, 32, 51 (pp. 113
 116, 120).
- X. § 4, 40 (p. 100); § 5, 67—70 (p. 104); § 6, 30 (p. 107)
 § 7, 29, 30 (p. 109); § 8, 22 (p. 111); § 9, 14, 33, 5
 (pp. 113, 116, 120).
- XI. § 4, 41 (p. 100); § 5, 71—74 (p. 104); § 6, 31, 32 (p. 107)
 § 7, 31 (p. 109); § 8, 23 (p. 111); § 9, 15, 34, 53 (pp. 114
 117, 121).
- XII. § 4, 42 (p. 100); § 5, 75—78 (p. 105); § 6, 33 (p. 107)
 § 7, 32, 33 (pp. 109, 110); § 8, 25 (p. 111); § 9, 16, 35
 54 (pp. 114, 117, 121).
- XIII. § 5, 79—83 (p. 105); § 6, 34, 35 (p. 107); § 7, 34 (p. 110)
 § 8, 26 (p. 111); § 9, 17, 36, 55 (pp. 114, 117, 121).
- XIV. § 5, 84—88 (p. 105); § 6, 36 (p. 107); § 7, 35, 36 (p. 110)
 § 8, 27 (p. 111); § 9, 18, 37, 56 (pp. 114, 117, 122).
- XV. § 5, 89—93 (p. 105); § 6, 37, 38 (p. 107); § 7, 37 (p. 110)
 § 8, 28 (p. 111); § 9, 19, 38, 57 (pp. 114, 118, 122).
- XVI. § 5, 94—97 (p. 106); § 6, 39 (p. 107); § 7, 38, 39 (p. 110)
 § 8, 29 (p. 111); § 9, 20, 39, 58 (pp. 114, 118, 122).
- XVII. § 5, 98—101 (p. 106); § 6, 40 (p. 107); § 7, 40 (p. 110)
 § 8, 30 (p. 111); § 9, 21, 41, 59, 60 (pp. 114, 118, 122, 123).
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BOOK VIII.

EXAMPLES, ANSWERS, AND SOLUTIONS.

[N.B. The numbers at the foot of each page indicate the pages where the corresponding matter may be found.]

CHAPTER I.

EXAMPLES.

§ 1.

Propositions of Relation, to be reduced to normal form.

1. I have been out for a walk.
2. I am feeling better.
3. No one has read the letter but John.
4. Neither you nor I are old.
5. No fat creatures run well.
6. None but the brave deserve the fair.
7. No one looks poetical unless he is pale.
8. Some persons, when contradicted, keep their temper.
9. I never neglect important business.
10. What is difficult needs attention.
11. What is unwholesome should be avoided.
12. All the laws passed last week relate to excise.
13. Logic puzzles me.
14. There are no Jews in the house.
15. Some dishes are unwholesome if not well-cooked.
16. Unexciting books make one drowsy.
17. When a man knows what he's about, he can detect a sharper.
18. You and I know what we're about.
19. Some bald people wear wigs.
20. Those who are fully occupied never talk about their grievances.
No riddles interest me if they can be solved.

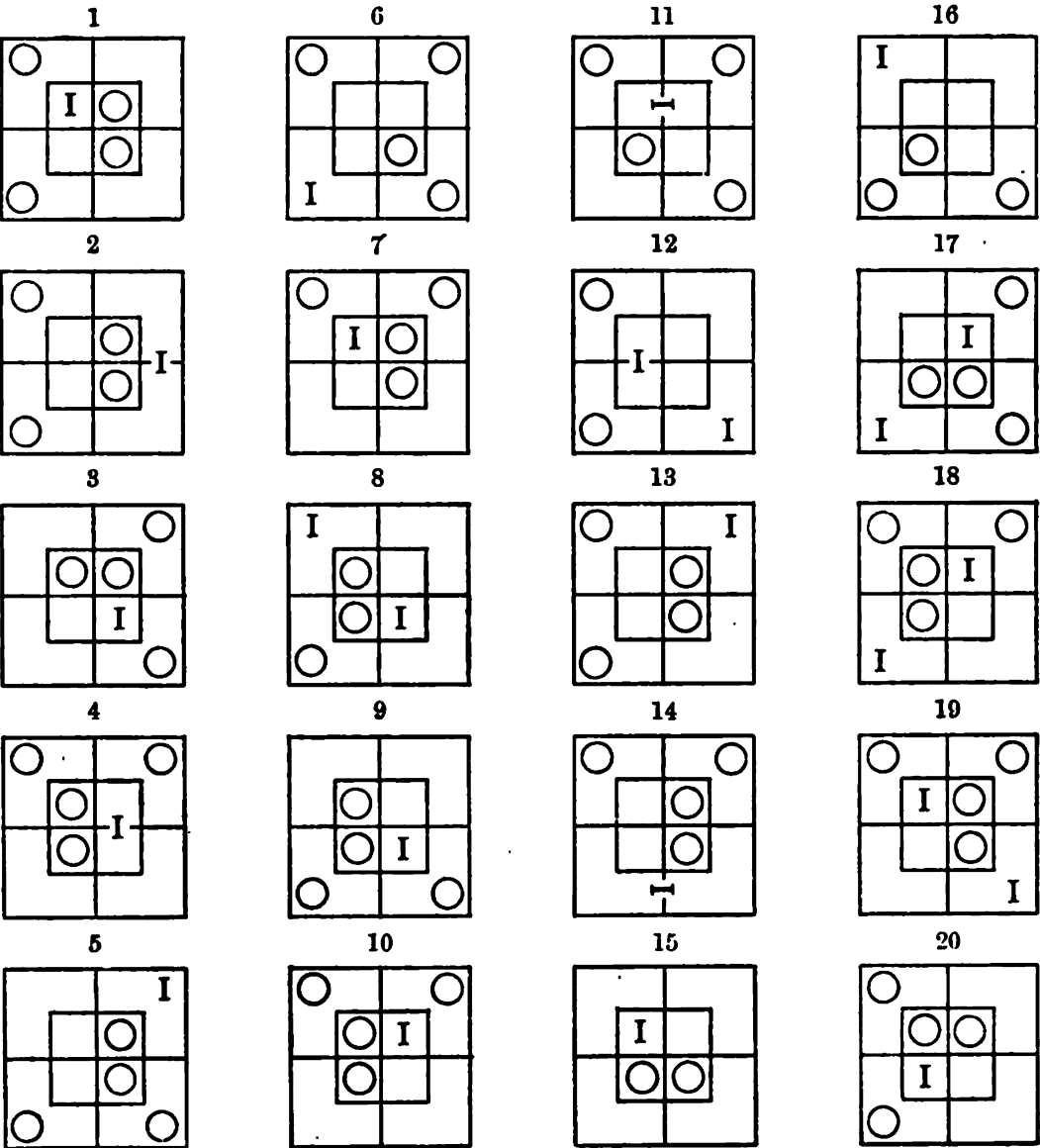
§ 2.

*Pairs of Abstract Propositions, to be represented
on the same Trilateral Diagram.*

- | | | |
|---|--|--|
| 1. No x are m ;
No m' are y . | 11. No x' are m' ;
No m are y . | 21. Some x' are m' ;
All y' are m . |
| 2. No x' are m' ;
All m' are y . | 12. Some x are m ;
All y' are m . | 22. No m are x ;
Some m are y . |
| 3. Some x' are m ;
No m are y . | 13. All x' are m ;
All m are y . | 23. No m' are x ;
All y are m' . |
| 4. All m are x ;
All m' are y' . | 14. Some x are m' ;
All m are y . | 24. All m are x ;
No y' are m' . |
| 5. All m' are x ;
All m' are y' . | 15. No m' are x' ;
All y are m . | 25. Some m are x ;
No y' are m . |
| 6. All x' are m' ;
No y' are m . | 16. All x are m' ;
No y are m . | 26. All m' are x' ;
Some y are m' . |
| 7. All x are m ;
All y' are m' . | 17. Some m' are x ;
No m' are y' . | 27. Some m are x' ;
No y' are m' . |
| 8. Some m' are x' ;
No m are y . | 18. All x are m' ;
Some m' are y' . | 28. No x are m' ;
All m are y' . |
| 9. All m are x' ;
No m are y . | 19. All m are x ;
Some m are y' . | 29. No x' are m ;
No m are y' . |
| 10. No m are x' ;
No y are m' . | 20. No x' are m ;
Some y are m . | 30. No x are m ;
Some y' are m' . |
| 31. Some m' are x ;
All y' are m . | 32. All x are m' ;
All y are m . | |
-

§ 3.

*Marked Trilateral Diagrams, to be interpreted
in terms of x and y .*



§ 4.

*Pairs of Abstract Propositions, proposed as Premisses :
Conclusions to be found.*

- | | | |
|--|--|--|
| 1. No m are x' ;
All m' are y . | 15. No x are m ;
No m' are y . | 29. No m are x ;
All y are m' . |
| 2. No m' are x ;
Some m' are y' . | 16. All x are m' ;
All y are m . | 30. All x are m ;
Some y are m' . |
| 3. All m' are x ;
All m' are y' . | 17. No x are m ;
All m' are y . | 31. All x are m ;
All y are m . |
| 4. No x' are m' ;
All y' are m . | 18. No x are m' ;
No m are y . | 32. No x are m' ;
All m are y . |
| 5. Some m are x' ;
No y are m . | 19. All m are x ;
All m are y' . | 33. No m are x ;
No m are y . |
| 6. No x' are m ;
No m are y . | 20. No m are x ;
All m' are y . | 34. No m are x' ;
Some y are m . |
| 7. No m are x' ;
Some y' are m . | 21. All x are m ;
Some m' are y ; | 35. No m are x ;
All y are m . |
| 8. All m' are x' ;
No m' are y . | 22. Some x are m ;
All y are m . | 36. All m are x' ;
Some y are m . |
| 9. Some x' are m' ;
No m are y' . | 23. All m are x ;
Some y are m . | 37. All m are x ;
No y are m . |
| 10. All x are m ;
All y' are m' . | 24. No x are m ;
All y are m . | 38. No m are x ;
No m' are y . |
| 11. No m are x ;
All y' are m' . | 25. Some m are x' ;
No m are y' . | 39. Some m are x ;
No m are y . |
| 12. No x are m ;
All y are m . | 26. No m are x' ;
All y are m . | 40. No x' are m ;
All y' are m . |
| 13. All m' are x ;
No y are m . | 27. All x are m' ;
All y' are m . | 41. All x are m' ;
No y are m' . |
| 14. All m are x ;
All m' are y . | 28. All m are x' ;
Some m are y . | 42. No m' are x ;
No y are m . |
-

§ 5

*Pairs of Concrete Propositions, proposed as Premisses :
Conclusions to be found.*

-
1. I have been out for a walk ;
I am feeling better.
 2. No one has read the letter but John ;
No one, who has *not* read it, knows what it is about.
 3. Those who are not old like walking ;
You and I are young.
 4. Your course is always honest ;
Your course is always the best policy.
 5. No fat creatures run well ;
Some greyhounds run well.
 6. Some, who deserve the fair, get their deserts ;
None but the brave deserve the fair.
 7. Some Jews are rich ;
All Esquimaux are Gentiles.
 8. Sugar-plums are sweet ;
Some sweet things are liked by children.
 9. John is in the house ;
Everybody in the house is ill.
 10. Umbrellas are useful on a journey ;
What is useless on a journey should be left behind.
 11. "Audible music causes vibration in the air ;"
"Inaudible music is not worth paying for."
 12. Some holidays are rainy ;
Rainy days are tiresome.
 13. No Frenchmen like plumpudding ;
All Englishmen like plumpudding.
 14. No portrait of a lady, that makes her simper or scowl, is satisfactory ;
No photograph of a lady ever fails to make her simper or scowl.
 15. All pale people are phlegmatic ;
No one looks poetical *unless* he is pale.
 16. No old misers are cheerful ;
Some old misers are *thin*.
 17. No one, when exercising self-control, fails to keep his temper ;
Some persons, when contradicted, ~~keep~~ ^{lose} their tempers.

Ans. 127 ; Sol. (1—12) 137—140 ; (13 &c) 146.]

18. All pigs are fat ;
Nothing that is fed on barley-water is fat.
19. All rabbits, that are not greedy, are black ;
No old rabbits are free from greediness.
20. Some pictures are not first attempts ;
No first attempts are really good.
21. I never neglect important business ;
Your business is unimportant.
22. Some lessons are difficult ;
What is difficult needs attention.
23. All clever people are popular ;
All obliging people are popular.
24. Thoughtless people do mischief ;
No thoughtful person forgets a promise.
25. Pigs cannot fly ;
Pigs are greedy.
26. All soldiers march well ;
Some babies are not soldiers.
27. No bride-cakes are wholesome
What is unwholesome should be avoided
28. John is industrious ;
No industrious people are unhappy.
29. No philosophers are conceited ;
Some conceited persons are not gamblers.
30. Some excise laws are unjust ;
All the laws passed last week relate to excise.
31. No military men write poetry ;
None of my lodgers are civilians.
32. No medicine is nice ;
Senna is a medicine.
33. Some circulars are not read with pleasure ;
No begging-letters are read with pleasure.
34. All Britons are brave ;
No sailors are cowards.
35. Nothing intelligible ever puzzles *me* ;
Logic puzzles me.
36. Some pigs are wild ;
All pigs are fat.

37. All wasps are unfriendly ;
All unfriendly creatures are unwelcome.
38. No old rabbits are greedy ;
All black rabbits are greedy.
39. Some eggs are hard-boiled ;
No eggs are uncrackable.
40. No antelope is ungraceful ;
Graceful creatures delight the eye.
41. All well-fed canaries sing loud ;
No canary is melancholy if it sings loud.
42. Some poetry is original ;
No original work is producible at will.
43. No country, that has been explored, is infested by dragons ;
Unexplored countries are fascinating.
44. No coals are white ;
No niggers are white.
45. No bridges are made of sugar ;
Some bridges are picturesque.
46. No children are patient ;
No impatient person can sit still.
47. No quadrupeds can whistle ;
Some cats are quadrupeds.
48. Bores are terrible ;
You are a bore.
49. Some oysters are silent ;
No silent creatures are amusing.
50. There are no Jews in the house ;
No Gentiles have beards a yard long.
51. Canaries, that do not sing loud, are unhappy ;
No well-fed canaries fail to sing loud.
52. All my sisters have colds ;
No one can sing who has a cold.
53. All that is made of gold is precious ;
Some caskets are precious.
54. Some buns are rich ;
All buns are nice.
55. All my cousins are unjust ;
All judges are just.

56. Pain is wearisome ;
No pain is eagerly wished for.
57. All medicine is nasty ;
Senna is a medicine.
58. Some unkind remarks are annoying ;
No critical remarks are kind.
59. No tall men have woolly hair ;
Niggers have woolly hair.
60. All philosophers are logical ;
An illogical man is always obstinate.
61. John is industrious ;
All industrious people are happy.
62. These dishes are all well-cooked ;
Some dishes are unwholesome if not well-cooked.
63. No exciting books suit feverish patients ;
Unexciting books make one drowsy.
64. No pigs can fly ;
All pigs are greedy.
65. When a man knows what he's about, he can detect a sharper ;
You and I know what we're about.
66. Some dreams are terrible ;
No lambs are terrible.
67. No bald creature needs a hairbrush ;
No lizards have hair.
68. All battles are noisy ;
What makes no noise may escape notice.
69. All my cousins are unjust ;
No judges are unjust.
70. All eggs can be cracked ;
Some eggs are hard-boiled.
71. Prejudiced persons are untrustworthy ;
Some unprejudiced persons are disliked.
72. No dictatorial person is popular ;
She is dictatorial.
73. Some bald people wear wigs ;
All your children have hair.
74. No lobsters are unreasonable ;
No reasonable creatures expect impossibilities.

75. No nightmare is pleasant ;
Unpleasant experiences are not eagerly desired.
76. No plumcakes are wholesome ;
Some wholesome things are nice.
77. Nothing that is nice need be shunned ;
Some kinds of jam are nice.
78. All ducks waddle ;
Nothing that waddles is graceful.
79. Sandwiches are satisfying ;
Nothing in this dish is unsatisfying.
80. No rich man begs in the street ;
Those who are not rich should keep accounts.
81. Spiders spin webs ;
Some creatures, that do not spin webs, are savage.
82. Some shops are not crowded ;
No crowded shops are comfortable.
83. Prudent travelers carry plenty of small change
Imprudent travelers lose their luggage.
84. Some geraniums are red ;
All these flowers are red.
85. None of my cousins are just ;
All judges are just.
86. No Jews are mad ;
All my lodgers are Jews.
87. Those who are fully occupied never talk about their grievances ;
Discontented people are always talking about their grievances.
88. None of my cousins are just ;
No judges are unjust.
89. All teetotalers like sugar ;
No nightingale drinks wine.
90. No riddles interest me if they can be solved ;
All these riddles are insoluble.
91. All clear explanations are satisfactory ;
Some excuses are unsatisfactory.
92. All elderly ladies are talkative ;
All good-tempered ladies are talkative.
93. No kind deed is unlawful ;
What is lawful may be done without scruple.

94. No babies are studious ;
No babies are good violinists.
95. All shillings are round ;
All these coins are round.
96. No honest men cheat ;
No dishonest men are trustworthy.
97. None of my boys are clever ;
None of my girls are greedy.
98. All jokes are meant to amuse ;
No Act of Parliament is a joke.
99. No eventful tour is ever forgotten ;
Uneventful tours are not worth writing a book about.
100. All my boys are disobedient ;
All my girls are discontented.
101. No unexpected pleasure annoys me ;
Your visit is an unexpected pleasure.

§ 6.

*Trios of Abstract Propositions, proposed as Syllogisms :
to be examined.*

- | | | |
|-------------------------|---------------------|----------------------|
| 1. Some x are m ; | No m are y' . | Some x are y . |
| 2. All x are m ; | No y are m' . | No y are x' . |
| 3. Some x are m' ; | All y' are m . | Some x are y . |
| 4. All x are m ; | No y are m . | All x are y' . |
| 5. Some m' are x' ; | No m' are y . | Some x' are y' . |
| 6. No x' are m ; | All y are m' . | All y are x' . |
| 7. Some m' are x' ; | All y' are m' . | Some x' are y' . |
| 8. No m' are x' ; | All y' are m' . | All y' are x . |
| 9. Some m are x' ; | No m are y . | Some x' are y' . |
| 10. All m' are x' ; | All m' are y . | Some y are x' . |
| 11. All x are m' ; | Some y are m . | Some y are x' . |
| 12. No x are m ; | No m' are y' . | No x are y' . |
| 13. No x are m ; | All y' are m . | All y' are x' . |
| 14. All m' are x' ; | All m' are y . | Some y are x' . |
| 15. Some m are x' ; | All y are m' . | Some x' are y' . |
| 16. No x' are m ; | All y' are m' . | Some y' are x . |
| 17. No m' are x ; | All m' are y' . | Some x' are y' . |

[Ans. 129, 130 ; Sol. (1—10) 140—141]

18. No x' are m ;	Some m are y .	Some x are y .
19. Some m are x' ;	All m are y .	Some y are x' .
20. No x' are m' ;	Some m' are y' .	Some x are y' .
21. No m are x ;	All m are y' .	Some x' are y' .
22. All x' are m ;	Some y are m' .	All x' are y' .
23. All m are x ;	No m' are y' .	No x' are y' .
24. All x are m' ;	All m' are y .	All x are y .
25. No x are m' ;	All m are y .	No x are y' .
26. All m are x' ;	All y are m .	All y are x' .
27. All x are m ;	No m are y' .	All x are y .
28. All x are m ;	No y' are m' .	All x are y .
29. No x' are m ;	No m' are y' .	No x' are y' .
30. All x are m ;	All m are y' .	All x are y' .
31. All x' are m' ;	No y' are m' .	All x' are y .
32. No x are m ;	No y' are m' .	No x are y' .
33. All m are x' ;	All y' are m .	All y' are x' .
34. All x are m' ;	Some y are m' .	Some y are x .
35. Some x are m ;	All m are y .	Some x are y .
36. All m are x' ;	All y are m .	All y are x' .
37. No m are x' ;	All m are y' .	Some x are y' .
38. No x are m ;	No m are y' .	No x are y' .
39. No m are x ;	Some m are y' .	Some x' are y' .
40. No m are x' ;	Some y are m .	Some x are y .

§ 7.

*Trios of Concrete Propositions, proposed as Syllogisms :
to be examined.*

1. No doctors are enthusiastic ;
You are enthusiastic.
You are not a doctor.
 2. All dictionaries are useful ;
No useful books should be taken as fuel.
Dictionaries should not be taken as fuel.
 3. No misers are unselfish.
None but misers save egg-shells.
No unselfish people save egg-shells.
 4. Some epicures are ungenerous ;
All my uncles are generous.
My uncles are not epicures.
- Ans. 130 ; Sol. (§7) 143, 144, 149.]

5. Gold is heavy ;
Nothing but gold will silence him.
Nothing light will silence him.
6. Some healthy people are fat ;
No unhealthy people are strong.
Some fat people are not strong.
7. "I saw it in a newspaper."
"All newspapers tell lies."
It was a lie.
8. Some cravats are not artistic ;
I admire anything artistic.
There are some cravats that I do not admire.
9. His songs never last an hour ;
A song, that lasts an hour, is tedious.
His songs are never tedious.
10. Some candles give very little light ;
Candles are *meant* to give light.
Some things, that are meant to give light, give very little.
11. All, who are anxious to learn, work hard ;
Some of these boys work hard.
Some of these boys are anxious to learn.
12. All lions are fierce ;
Some lions do not drink coffee.
Some creatures that drink coffee are not fierce.
13. No misers are generous ;
Some old men are ungenerous.
Some old men are misers.
14. No fossil can be crossed in love ;
An oyster may be crossed in love.
Oysters are not fossils.
15. All uneducated people are shallow ;
Students are all educated.
No students are shallow.
16. All young lambs jump ;
No young animals are healthy, unless they jump.
All young lambs are healthy.
17. Ill-managed business is unprofitable ;
Railways are never ill-managed.
All railways are profitable.
18. No Professors are ignorant ;
All ignorant people are vain.
No professors are vain.

19. A prudent man shuns hyænas ;
~~Anything but gold~~ will fail to silence him.
No banker fails to shun hyænas.
20. All wasps are unfriendly ;
No puppies are unfriendly.
Puppies are not wasps.
21. No Jews are honest ;
Some Gentiles are rich.
Some rich people are dishonest.
22. No idlers win fame ;
Some painters are not idle.
Some painters win fame.
23. No monkeys are soldiers ;
All monkeys are mischievous.
Some mischievous creatures are not soldiers.
24. All these bonbons are chocolate-creams ;
All these bonbons are delicious.
Chocolate-creams are delicious.
25. No muffins are wholesome ;
All buns are unwholesome.
Buns are not muffins.
26. Some unauthorised reports are false ;
All authorised reports are trustworthy.
Some false reports are not trustworthy.
27. Some pillows are soft ;
No pokers are soft.
Some pokers are not pillows.
28. Improbable stories are not easily believed ;
None of his stories are probable.
None of his stories are easily believed.
29. No thieves are honest ;
Some dishonest people are found out.
Some thieves are found out.
30. No muffins are wholesome ;
All puffy food is unwholesome.
All muffins are puffy.
31. No birds, except peacocks, are proud of their tails ;
Some birds, that are proud of their tails, cannot sing.
Some peacocks cannot sing.
32. Warmth relieves pain ;
Nothing, that does not relieve pain, is useful in toothache.
Warmth is useful in toothache.

33. No bankrupts are rich ;
Some merchants are not bankrupts.
Some merchants are rich.
34. Bores are dreaded ;
No bore is ever begged to prolong his visit.
No one, who is dreaded, is ever begged to prolong his visit
35. All wise men walk on their feet ;
All unwise men walk on their hands.
No man walks on both.
36. No wheelbarrows are comfortable ;
No uncomfortable vehicles are popular.
No wheelbarrows are popular.
37. No frogs are poetical ;
Some ducks are unpoetical.
Some ducks are not frogs.
38. No emperors are dentists ;
All dentists are dreaded by children.
No emperors are dreaded by children.
39. Sugar is sweet ;
Salt is not sweet.
Salt is not sugar.
40. Every eagle can fly ;
Some pigs cannot fly.
Some pigs are not eagles.

§ 8.

*Sets of Abstract Propositions, proposed as Premisses
for Sorites : Conclusions to be found.*

[N.B. At the end of this Section instructions are given for varying these Examples.]

- | | | | |
|----------------------|----------------------|------------------------|----------------------|
| 1. | 3. | 5. | 7. |
| 1. No c are d ; | 1. No b are a ; | 1. All b' are a' ; | 1. No d are b' ; |
| 2. All a are d ; | 2. No c are d' ; | 2. No b are c ; | 2. All b are a ; |
| 3. All b are c . | 3. All d are b . | 3. No a' are d . | 3. No c are d' . |
| 2. | 4. | 6. | 8. |
| 1. All d are b ; | 1. No b are c ; | 1. All a are b' ; | 1. No b' are d ; |
| 2. No a are c' ; | 2. All a are b ; | 2. No b' are c ; | 2. No a' are b ; |
| 3. No b are c . | 3. No c' are d . | 3. All d are a . | 3. All c are d . |

[Ans. 130, 131 ; Sol. 153—155.]

9.

1. All b' are a ;
2. No a are d ;
3. All b are c .

10.

1. No c are d ;
2. All b are c ;
3. No a are d' .

11.

1. No b are c ;
2. All d are a ;
3. All c' are a' .

12.

1. No c are b' ;
2. All c' are d' ;
3. All b are a .

13.

1. All d are e ;
2. All c are a ;
3. No b are d' ;
4. All e are a' .

14.

1. All c are b ;
2. All a are e ;
3. All d are b' ;
4. All a' are c .

15.

1. No b' are d ;
2. All e are c ;
3. All b are a ;
4. All d' are c' .

16.

1. No a' are e ;
2. All d are c' ;
3. All a are b ;
4. All e' are d .

17.

1. All d are c ;
2. All a are e ;
3. No b are d' ;
4. All c are e' .

18.

1. All a are b ;
2. All d are e ;
3. All a' are c' ;
4. No b are e .

19.

1. No b are c ;
2. All e are h ;
3. All a are b ;
4. No d are h ;
5. All e' are c .

20.

1. No d are h' ;
2. No c are e ;
3. All h are b ;
4. No a are d' .
5. No b are e' .

21.

1. All b are a ;
2. No d are h ;
3. No c are e ;
4. No a are h' ;
5. All c' are b .

22.

1. All e are d' ;
2. No b' are h' ;
3. All c' are d ;
4. All a are e ;
5. No c are h .

23.

1. All b' are a' ;
2. No d are e' ;
3. All h are b' ;
4. No c are e ;
5. All d' are a .

24.

1. All h' are k' ;
2. No b' are a ;
3. All c are d ;
4. All e are h' ;
5. No d are k' ;
6. No b are c' .

25.

1. All a are d ;
2. All k are b ;
3. All e are h ;
4. No a' are b ;
5. All d are c ;
6. All h are k .

26.

1. All a' are h ;
2. No d' are k' ;
3. All e are b' ;
4. No h are k ;
5. All a are c ;
6. No b' are d .

27.

1. All e are d' ;
2. No h are b ;
3. All a' are k ;
4. No c are e' ;
5. All b' are d ;
6. No a are c' .

28.

1. No a' are k ;
2. All e are b ;
3. No h are k' ;
4. No d' are c ;
5. No a are b ;
6. All c' are h .

29.

1. No e are k ;
2. No b' are m ;
3. No a are c' ;
4. All h' are e ;
5. All d are k ;
6. No c are b ;
7. All d' are l ;
8. No h are m' .

30.

1. All n are m ;
2. All a' are e ;
3. No c' are l ;
4. All k are r' ;
5. No a are h' ;
6. No d are l' ;
7. No c are n' ;
8. All e are b ;
9. All m are r ;
10. All h are d .

[N.B. In each Example, in Sections 8 and 9, it is possible to begin with *any* Premiss, at pleasure, and thus to get as many different Solutions (all of course yielding the *same* Complete Conclusion) as there are Premisses in the Example. Hence § 8 really contains 129 different Examples, and § 9 contains 273.]

§ 9.

Sets of Concrete Propositions, proposed as Premisses for Sorites: Conclusions to be found.

1.

- (1) Babies are illogical ;
- (2) Nobody is despised who can manage a crocodile ;
- (3) Illogical persons are despised.

Univ. "persons" ;

a = able to manage &c ; b = babies ; c = despised ; d = logical.

2.

- (1) My saucepans are the only things I have that are made of tin ;
- (2) I find all *your* presents very useful ;
- (3) None of my saucepans are of the slightest use.

Univ. "things of mine" ;

a = made of tin ; b = my saucepans ; c = useful ; d = your presents.

3.

- (1) No potatoes in this house, that are new, have been boiled ;
- (2) All the potatoes in this dish are fit to eat ;
- (3) No unboiled potatoes are fit to eat.

Univ. "potatoes in this house" ;

a = boiled ; b = eatable ; c = in this dish ; d = new.

4.

- (1) There are no Jews in the kitchen ;
- (2) No Gentiles say "shpoonj" ;
- (3) My servants are all in the kitchen.

Univ. "persons" ;

a = in kitchen ; b = Jews ; c = my servants ; d = who say &c.

5.

- (1) No ducks waltz ;
- (2) No officers ever decline to waltz ;
- (3) All my poultry are ducks.

Univ. "creatures" ;

a = ducks ; b = my poultry ; c = officers ; d = willing to waltz.

6.

- (1) Any one who is sane can do Logic ;
- (2) No lunatics are fit to serve on a jury ;
- (3) None of *your* sons can do Logic.

Univ. "persons" ;

a = able to do Logic ; b = fit to serve &c ; c = sane ; d = your sons.

7.

- (1) There are no pencils of mine in this box ;
- (2) No sugar-plums are cigars ;
- (3) The whole of my property, that is not in this box, consists of cigars.

Univ. "things of mine" ;

a = cigars ; b = in this box ; c = pencils ; d = sugar-plums.

[Ans. 131 ; Sol. 156, 157.]

8.

- (1) No experienced person is incompetent ;
- (2) Jenkins is always blundering ;
- (3) No competent person is always blundering.

Univ. "persons" ;

 a = always blundering ; b = competent ; c = experienced ; d = Jenkins.

9.

- (1) No terriers wander among the signs of the zodiac ;
- (2) Nothing that does not wander among the signs of the zodiac, is a
- (3) Nothing but a terrier has a curly tail. [comet ;

Univ. "things" ;

 a = comets ; b = curly-tailed ; c = terriers ; d = wandering &c.

10.

- (1) No one takes in the *Times*, unless he is well-educated ;
- (2) No hedge-hogs can read ;
- (3) Those who cannot read are not well-educated.

Univ. "creatures" ;

 a = can read ; b = hedge-hogs ; c = taking in &c ; d = well-educated.

11.

- (1) All puddings are nice ;
- (2) This dish is a pudding ;
- (3) No nice things are wholesome.

Univ. "things" ;

 a = nice ; b = puddings ; c = this dish ; d = wholesome.

12.

- (1) Showy talkers think too much of themselves ;
- (2) No really well-informed people are bad company ;
- (3) People who think too much of themselves are not good company.

Univ. "persons" ;

 a = good company ; b = really well &c ; c = showy &c ; d = thinking &c.

13.

- (1) All humming-birds are richly coloured ;
- (2) No large birds live on honey ;
- (3) Birds that do not live on honey are dull in colour.

Univ. "birds" ;

 a = humming-birds ; b = large ; c = living on &c ; d = richly coloured.

14.

- (1) No Gentiles have hooked noses ;
- (2) A man who is a good hand at a bargain always makes money ;
- (3) No Jew is ever a bad hand at a bargain.

Univ. "persons" ;

 a = good hands &c ; b = hook-nosed ; c = Jews ; d = making money.

Ans. 131 ; Sol. 157, 158.]

15.

- (1) No acrobatic feats, that are not announced in the bills of a circus, are ever attempted there ;
- (2) No acrobatic feat is possible, if it involves turning a quadruple somersault ;
- (3) No impossible feat is ever announced in the bills of a circus.

Univ. "acrobatic feats" ;

a = announced & c ; b = attempted & c ; c = involving & c ; d = possible.

16.

- (1) My gardener is well worth listening to on military subjects ;
- (2) No one can remember the battle of Waterloo, unless he is very old ;
- (3) Nobody is really worth listening to on military subjects, unless he can remember the battle of Waterloo.

Univ. "persons" ;

a = can remember & c ; b = my gardener ; c = well worth & c ; d = very old.

17.

- (1) All ducks in this village, that are branded 'B,' belong to Mrs. Bond ;
- (2) Ducks in this village never wear lace collars, unless they are branded
- (3) Mrs. Bond has no gray ducks. ['B' ;

Univ. "ducks in this village" ;

a = belonging to & c ; b = branded 'B' ; c = gray ; d = wearing & c .

18.

- (1) All the M.P.'s, who belong to the House of Commons, have perfect self-command ;
- (2) No M.P., who wears a coronet, should ride in a donkey-race ;
- (3) All the M.P.'s, who belong to the House of Lords, wear coronets.

Univ. "M.P.'s" ;

a = belonging to House of Commons ; b = having perfect & c ; c = may ride & c ;
 d = wearing & c .

19.

- (1) All the old articles in this cupboard are cracked ;
- (2) No jugs in this cupboard are new ;
- (3) Nothing in this cupboard, that is cracked, will hold water.

Univ. "things in this cupboard" ;

a = cracked ; b = jugs ; c = old ; d = will hold water.

20.

- (1) Nobody, who really appreciates Beethoven, fails to keep silence while the Moonlight Sonata is being played ;
- (2) Guinea-pigs are hopelessly ignorant of music ;
- (3) No one, who is hopelessly ignorant of music, ever keeps silence while the Moonlight Sonata is being played.

Univ. "creatures" ;

a = guinea-pigs ; b = hopelessly & c ; c = keeping & c ; d = really & c .

21.

- (1) Puppies, that will not lie still, are always grateful for the loan of a skipping-rope ;
- (2) A lame puppy would not say "thank you" if you offered to lend it a skipping-rope.

- (3) None but lame puppies ever care to do worsted-work.

Univ. "puppies";

a = caring to &c ; b = grateful &c ; c = lame ; d = that will lie &c.

22.

- (1) All unripe fruit is unwholesome ;
 (2) All these apples are wholesome ;
 (3) No fruit, grown in the shade, is ripe.

Univ. "fruit" ;

a = grown in shade ; b = ripe ; c = these apples ; d = wholesome.

23.

- (1) No goods in this shop, that have been bought and paid for, are still on sale ;
 (2) None of the goods may be carried away, unless labeled "sold" ;
 (3) None of the goods are labeled "sold," unless they have been bought and paid for.

Univ. "goods in this shop" ;

a = bought &c ; b = labeled &c ; c = may be carried &c ; d = on sale.

24.

- (1) Coloured flowers are always scented ;
 (2) I dislike flowers that are not grown in the open air ;
 (3) No flowers grown in the open air are colourless.

Univ. "flowers" ;

a = coloured ; b = grown in &c ; c = liked by me ; d = scented.

25.

- (1) None of the names in this list are unsuitable for the hero of a
 (2) Names beginning with a vowel are always melodious ; [romance ;
 (3) No name is suitable for the hero of a romance, if it begins with a consonant.

Univ. "names" ;

a = beginning with vowel ; b = in this list ; c = melodious ; d = suitable &c.

26.

- (1) Things sold in the street are of no great value ;
 (2) Nothing but rubbish can be had for a song ;
 (3) Eggs of the Great Awk are very valuable ;
 (4) It is only what is sold in the street that is really *rubbish*.

Univ. "things" ;

a = can be had for &c ; b = eggs of &c ; c = rubbish ; d = sold in street ;
 e = very valuable.

27.

- (1) No books sold here have gilt edges, except what are in the front shop ;
 (2) All the *authorised* editions have red labels ;
 (3) All the books with red labels are priced at 5s. and upwards ;
 (4) None but *authorised* editions are ever placed in the front shop.

Univ. "books sold here" ;

a = authorised ; b = gilt-edged ; c = in front shop ; d = labeled &c ;
 e = priced at &c.

28.

- (1) Any remedy for bleeding, which fails to check it, is a mockery ;
- (2) Tincture of Calendula is not to be despised ;
- (3) When you cut your finger, you will find *any* remedy useful, which
- (4) All mockeries are despicable. [will check the bleeding.

Univ. "remedies for bleeding" ;

a = despicable ; b = mockeries ; c = Tincture of &c ; d = useful when &c.
 e = will check bleeding.

29.

- (1) All my sons are slim ;
- (2) Nobody is healthy who takes no exercise ;
- (3) Gluttons are always fat ;
- (4) No daughter of mine takes any exercise.

Univ. "my children" ;

a = fat ; b = gluttons ; c = healthy ; d = sons ; e = taking &c.

30.

- (1) The only books in this library, that I do *not* recommend for reading,
are unhealthy in tone ;
- (2) The bound books are all well-written ;
- (3) All the romances are healthy in tone ;
- (4) I do not recommend you to read any of the unbound books.

Univ. "books in this library" ;

a = bound ; b = healthy &c ; c = recommended &c ; d = romances ;
 e = well-written.

31.

- (1) No birds, except ostriches, are 9 feet high ;
- (2) There are no birds in this aviary that belong to any one but *me* ;
- (3) No ostrich lives on mince-pies ;
- (4) I have no birds less than 9 feet high.

Univ. "birds" ;

a = in this &c ; b = living on &c ; c = my ; d = 9 feet &c ; e = ostriches.

32.

- (1) A plum-pudding, that is not really solid, is mere porridge ;
- (2) Any plum-pudding, served at my table, has been boiled in a cloth ;
- (3) A plum-pudding that is mere porridge is indistinguishable from soup ;
- (4) No plum-puddings are really solid, except what are served at *my*
table.

Univ. "plum-puddings" ;

a = boiled in &c ; b = distinguishable from &c ; c = mere porridge ;
 d = really solid ; e = served at my table.

33.

- (1) No boys under 12 are admitted to this school as boarders ;
- (2) All the industrious boys have red hair ;
- (3) None of the day-boys learn Greek ;

[Ans. 132 ; Sol. 159.

(4) None but those under 12 are idle.

Univ. "boys in this school";

a = boarders ; b = industrious ; c = learning & c ; d = red-haired ; e = under 12.

34.

(1) The only articles of food, that my doctor allows me, are such as are not very rich ;

(2) Nothing that agrees with me is unsuitable for supper ;

(3) Wedding-cake is always very rich ;

(4) My doctor allows me any article of food that is suitable for supper.

Univ. "articles of food" ;

a = agreeing with & c ; b = allowed by & c ; c = suitable for & c ;
 d = very rich ; e = wedding-cake.

35.

(1) None of the things, usually met with at sea, are mermaids ;

(2) Anything entered in the log, as met with at sea, is sure to be worth remembering ;

(3) *I* have never met with anything worth remembering, when on a voyage ;

(4) Anything met with at sea, that is unusual, is sure to be recorded in the log ;

Univ. "things met with at sea" ;

a = entered in log ; b = mermaids ; c = met with by me ; d = usual ;
 e = worth remembering.

36.

(1) No discussions are likely to rouse the British Lion, so long as they are checked when they become too noisy ;

(2) The peacefulness of our Debating-Club is endangered by any discussion that is not wisely conducted ;

(3) The British Lion is liable to be roused by any discussion that goes on while Tomkins is in the Chair ;

(4) A discussion, that is wisely conducted, is always checked when it becomes too noisy.

Univ. "discussions in our Debating-Club" ;

a = checked when & c ; b = dangerous to & c ; c = going on while & c ;
 d = likely to rouse & c ; e = wisely conducted.

37.

(1) All the Eton men in this College play cricket ;

(2) None but the Scholars dine at the higher table ;

(3) None of the cricketers row ;

(4) *My* friends all come from Eton ;

(5) All the Scholars are rowing-men.

Univ. "men in this College" ;

a = cricketers ; b = dining at & c ; c = Etonians ; d = my friends ;
 e = rowing-men ; h = Scholars.

Ans. 132 ; Sol. 160.] .

38.

- (1) I despise anything that cannot be used as a bridge ;
- (2) Nothing, that is worth writing an ode to, would be an unwelcome gift to me ;
- (3) A rainbow will not bear the weight of a wheel-barrow ;
- (4) Whatever can be used as a bridge will bear the weight of a wheel-
- (5) I would not take, as a gift, a thing that I despise. [barrow ; Univ. "things" ;

a = acceptable to me ; b = despised by me ; c = rainbows ; d = useful as a bridge ; e = will bear & c ; h = worth writing & c .

39.

- (1) Promise-breakers are untrustworthy ;
- (2) Wine-drinkers are very communicative ;
- (3) A man who keeps his promises is honest ;
- (4) No teetotalers are pawnbrokers ;
- (5) One can always trust a very communicative person.

Univ. "persons" ;

a = honest ; b = pawnbrokers ; c = promise-keepers ; d = trustworthy ;
 e = very communicative ; h = wine-drinkers.

40.

- (1) No kitten, that loves fish, is unteachable ;
- (2) No kitten without a tail will play with a gorilla ;
- (3) Kittens with whiskers always love fish ;
- (4) No teachable kitten has green eyes ;
- (5) No kittens have tails unless they have whiskers.

Univ. "kittens" ;

a = fish-loving ; b = green-eyed ; c = tailed ; d = teachable ; e = whiskered ;
 h = will play with a gorilla.

41.

- (1) No interesting poems are unpopular among people of real taste ;
- (2) No modern poetry is free from affectation ;
- (3) All *your* poems are on the subject of soap-bubbles ;
- (4) No affected poetry is popular among people of real taste ;
- (5) No ancient poem is on the subject of soap-bubbles.

Univ. "poems" ;

a = affected ; b = ancient ; c = interesting ; d = on the subject of & c ;
 e = popular among & c ; h = your.

42.

- (1) All the fruit at this Show, that fails to get a prize, is the property of
- (2) None of my peaches have got prizes ; [the Committee ;
- (3) None of the fruit, sold off in the evening, is unripe ;
- (4) None of the ripe fruit has been grown in a hot-house ;
- (5) All fruit, that belongs to the Committee, is sold off in the evening.

Univ. "fruit at this show" ;

a = belonging to & c ; b = getting prize ; c = grown in & c ; d = my peaches ;
 e = ripe ; h = sold off & c .

43.

- (1) Gentiles have no objection to pork ;
 - (2) Nobody who admires pigsties ever reads Hogg's poems ;
 - (3) No Mandarin can think in Hebrew ;
 - (4) Any one, who does not object to pork, admires pigsties ;
 - (5) No Jew is unable to think in Hebrew.
- Univ. "persons" ; a = admiring &c ; b = can think in &c ; c = Jews ;
 d = Mandarins ; e = objecting to &c ; h = reading &c.

44.

- (1) There is no box of mine in this room that I dare open ;
 - (2) My writing-desk is made of rose-wood ;
 - (3) All my boxes are painted, except those in this room ;
 - (4) There is no box of mine that I dare not open, unless it is full of live
 - (5) All my rose-wood boxes are unpainted. [scorpions ;
- Univ. "my boxes" ; a = full of &c ; b = in this room ; c = painted ;
 d = rose-wood ; e = that I dare open ; h = writing-desks.

45.

- (1) All writers, who understand human nature, are clever ;
 - (2) No one is a true poet unless he can stir the hearts of men ;
 - (3) Shakespeare wrote "Hamlet" ;
 - (4) None but those who understand human nature can stir the hearts of
 - (5) None but a true poet could have written "Hamlet." [men ;
- Univ. "writers" ; a = can stir &c ; b = clever ; c = Shakespeare ;
 d = true poets ; e = understanding &c ; h = wrote 'Hamlet' ;

46.

- (1) When I work a Logic-example without grumbling, you may be sure it is one that I can understand ;
 - (2) These Sorites are not arranged in regular order, like the examples 1
 - (3) No easy example ever makes my head ache ; [am used to ;
 - (4) I ca'n't understand examples that are not arranged in regular order, like those I am used to ;
 - (5) I never grumble at an example, unless it gives me a headache.
- Univ. "Logic-examples worked by me" ;
 a = arranged in &c ; b = easy ; c = grumbled at by me ; d = making my head ache ; e = these Sorites ; h = understood by me.

47.

- (1) Animals, that do not kick, are always unexcitable ;
 - (2) Donkeys have no horns ;
 - (3) A buffalo can always toss one over a gate ;
 - (4) No animals that kick are easy to swallow ;
 - (5) No hornless animal can toss one over a gate ;
 - (6) All animals are excitable, except buffaloes.
- Univ. "animals" ; a = buffaloes ; b = can toss &c ; c = donkeys ;
 d = easy to swallow ; e = excitable ; h = horned ; k = kicking.

Ans. 132 ; Sol. 160, 161.]

48.

- (1) No one, who is going to a party, ever fails to brush his hair ;
- (2) No one looks fascinating, if he is untidy ;
- (3) Opium-eaters have no self-command ;
- (4) Any one, who has brushed his hair, looks fascinating ;
- (5) No one wears white kid gloves, unless he is going to a party ;
- (6) A man is always untidy, if he has no self-command.

Univ. "persons" ;

a = going to &c ; b = hair-brushing ; c = looks fascinating ; d = opium-eaters ; e = self-commanding ; h = tidy ; k = wearing white &c.

49.

- (1) None of the pictures here, except the battle-pieces, are valuable ;
- (2) None of the unframed ones are varnished ;
- (3) All the battle-pieces are painted in oils ;
- (4) All those that have been sold are valuable ;
- (5) All the English ones are varnished ;
- (6) All those in frames have been sold.

Univ. "the pictures here" ;

a = battle-pieces ; b = English ; c = framed ; d = oil-paintings ; e = sold ;
 h = valuable ; k = varnished.

50.

- (1) I can trust any animal that belongs to me ;
- (2) Dogs gnaw bones ;
- (3) I admit no animals into my study, unless they will beg when told
- (4) All the animals in the yard are mine ; [to do so ;
- (5) I admit any animal, that I can trust, into my study ;
- (6) The only animals, that will beg when told to do so, are dogs.

Univ. "animals" ;

a = admitted to &c ; b = bone-gnawing ; c = dogs ; d = in the yard ;
 e = my ; h = that I can trust ; k = will beg when told.

51.

- (1) Anything, that is not absolutely ugly, may be kept in a drawing-
- (2) Nothing, that is encrusted with salt, is ever quite dry ; [room ;
- (3) Nothing should be kept in a drawing-room, unless it is free from damp ;
- (4) Bathing-machines are always kept near the sea ;
- (5) Nothing, that is made of mother-of-pearl, can be absolutely ugly ;
- (6) Whatever is kept near the sea gets encrusted with salt.

Univ. "things" ;

a = absolutely ugly ; b = bathing-machines ; c = encrusted &c ; d = kept near sea ; e = made of &c ; h = may be kept in &c ; k = quite dry.

52.

- (1) No husband, who is always giving his wife new dresses, can be a cross-grained man ;
- (2) A methodical husband always comes home for his tea ;
- (3) No one, who hangs up his hat on the gas-jet, can be a man that is kept in proper order by his wife ;
- (4) A good husband is always giving his wife new dresses ;

[Ans. 132 ; Sol. 161.]

(5) No husband can fail to be cross-grained, if his wife does not keep him in proper order ;

(6) An unmethodical husband always hangs up his hat on the gas-jet.

Univ. "husbands" ;

a = always giving &c ; b = comes home &c ; c = cross-grained ; d = good ;

e = hangs up &c ; h = kept in &c ; k = methodical.

53.

(1) I call no day "unlucky," when Robinson is civil to me ;

(2) Wednesdays are always cloudy ;

(3) When people take umbrellas, the day seldom turns out fine ;

(4) The only days when Robinson is uncivil to me are Wednesdays ;

(5) Nobody leaves his umbrella behind him when it is raining ;

(6) My "lucky" days usually turn out fine.

Univ. "days" ;

a = called by me 'lucky' ; b = cloudy ; c = rainy ; d = usually turns &c ;

e = Wednesdays ; h = when people take &c ; k = when Robinson &c.

54.

(1) I never put a cheque on that file, unless I am anxious about it ;

(2) All the cheques received this year, that are not marked with a cross, are payable to bearer ;

(3) No cheques are ever brought back to me, unless they are dishonoured ;

(4) All the cheques received this year, that are marked with a cross, are for amounts of over £100 ;

(5) All the cheques received this year, that are not on that file, are marked "not negotiable" ;

(6) No cheque of yours has ever been dishonoured ;

(7) I am never anxious about a cheque, unless it is brought back to me ;

(8) None of the cheques received this year, that are marked "not negotiable," are for amounts of over £100.

Univ. "cheques received this year" ;

a = brought &c ; b = honoured ; c = marked with cross ; d = marked 'not negotiable' ; e = on that file ; h = over £100 ; k = payable to &c ;

l = that I am anxious about ; m = your.

55.

(1) All the dated letters in this room are written on blue paper ;

(2) None of them are in black ink, except those that are written in the

(3) I have not destroyed any that I can read ; [third person ;

(4) None of those on one sheet are undated ;

(5) All, that are not crossed, are in black ink ;

(6) All Brown's letters begin with "Dear Sir" ;

(7) Those written on blue paper are all destroyed ;

(8) No letter, on more than one sheet, is crossed ;

(9) No letter, that begins with "Dear Sir," is written in the third person.

Univ. "letters in this room" ;

a = beginning &c ; b = Brown's ; c = crossed ; d = dated ; e = destroyed ;

h = in black ink ; k = in third person ; l = on blue paper ; m = on one sheet ; n = that I can read.

56.

- (1) No shark ever doubts that it is well fitted out ;
- (2) A fish, that cannot dance a minuet, is contemptible ;
- (3) No fish is quite certain that it is well fitted out, unless it has three
- (4) All fishes, except sharks, are kind to children. [rows of teeth ;
- (5) No heavy fish can dance a minuet ;
- (6) A fish with three rows of teeth is not to be despised.

Univ. "fishes" ;

a = can dance &c ; b = certain that he is &c ; c = contemptible ;
 d = has 3 rows &c ; e = heavy ; h = kind to &c ; k = sharks.

57.

- (1) All the human race, except my footmen, have a certain amount of common-sense ;
- (2) No one, who lives on barley-sugar, can be anything but a mere baby ;
- (3) None but a hop-scotch player knows what real happiness is ;
- (4) No mere baby has a grain of common sense ;
- (5) No engine-driver ever plays hop-scotch ;
- (6) No footman of mine is ignorant of what true happiness is ;

Univ. "human beings" ;

a = engine-drivers ; b = has common sense ; c = hop-scotch players ;
 d = knows what &c ; e = living on &c ; h = mere babies ;
 k = my footmen.

58.

- (1) Any idea of mine, that cannot be expressed as a Syllogism, is really ridiculous ;
- (2) None of my ideas about Bath-buns are worth writing down ;
- (3) No idea, that fails to come true, can be expressed as a Syllogism ;
- (4) I never have any really ridiculous idea, that I do not at once refer
- (5) My dreams are all about Bath-buns ; [to my solicitor ;
- (6) I never refer anything to my solicitor, unless it is worth writing down.

Univ. "my ideas" ;

a = about Bath-buns ; b = can be expressed &c ; c = comes true ;
 d = dreams ; e = really ridiculous ; h = referred to my solicitor ;
 k = worth writing down.

59.

- (1) Animals are always mortally offended if I fail to notice them ;
- (2) The only animals that belong to me are in that field ;
- (3) No animal can guess a conundrum, unless it has been properly trained
- (4) None of the animals in that field are badgers ; [in a Board-School ;
- (5) When an animal is mortally offended, it rushes about wildly and
- (6) I never notice any animal, unless it belongs to me ; [howls.
- (7) No animal, that has been properly trained in a Board-School, ever rushes about wildly and howls.

Univ. "animals" ;

a = badgers ; b = can guess &c ; c = in that field ; d = mortally offended &c ;
 e = my ; h = noticed by me ; k = properly &c ; l = rushes about &c.

[Ans. 132 ; Sol. 162.

60.

- (1) The only animals in this house are cats ;
- (2) Any animal is suitable for a pet, if it loves to gaze at the moon ;
- (3) When I detest an animal, I avoid it ;
- (4) No animals are carnivorous, unless they prowl at night ;
- (5) No cat fails to kill mice ;
- (6) No animals ever take to me, except what are in this house ;
- (7) Kangaroos are not suitable for pets ;
- (8) None but carnivora kill mice ;
- (9) I detest an animal that does not take to me ;
- (10) Animals, that prowl at night, always love to gaze at the moon.

Univ. "animals" ;

a = avoided by me ; b = carnivora ; c = cats ; d = detested by me ;
 e = in this house ; h = kangaroos ; k = killing mice ; l = loving to &c ;
 m = prowling &c ; n = suitable &c ; r = taking to me.

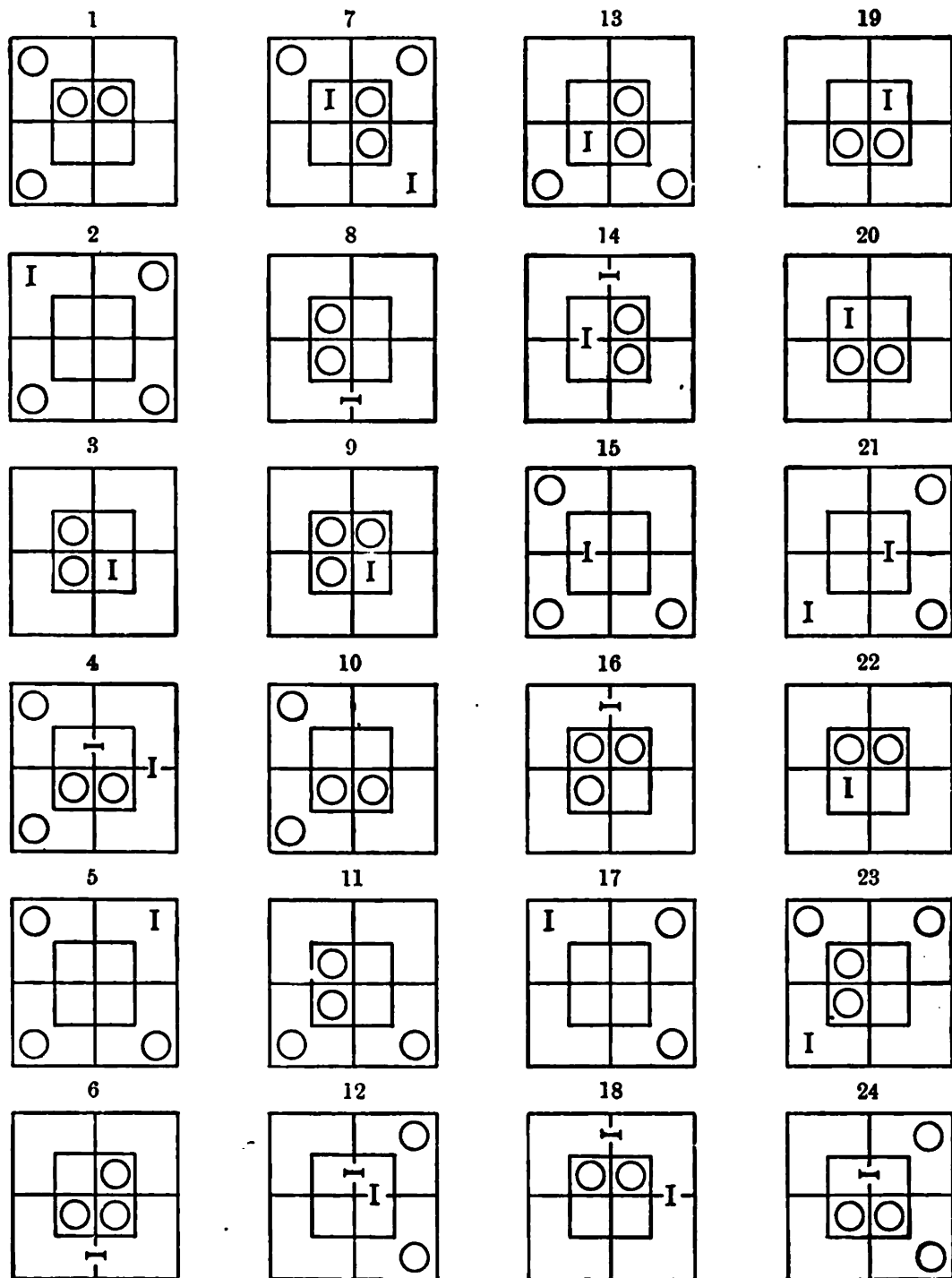
Ans. 132 ; Sol. 162.]

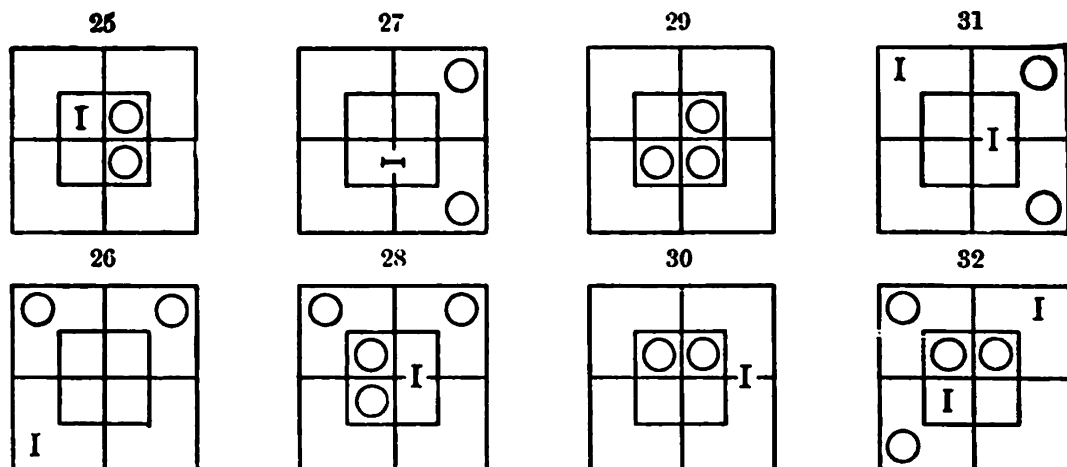
CHAPTER II.

ANSWERS.

Answers to § 1.

1. "All" *Sign of Quantity.*
"persons represented by the Name 'I'" (or "I's") *Subject.*
"are" *Copula.*
"persons who have been out for a walk" *Predicate.*
- or, more briefly,
"All | 'I's' | are | persons who have been out for a walk".
2. "All | 'I's' | are | persons who feel better".
3. "No | persons who are not 'John' | are | persons who have read the letter".
4. "No | Members of the Class 'you and I' | are | old persons".
5. "No | fat creatures | are | creatures that run well".
6. "No | not-brave persons | are | persons deserving of the fair".
7. "No | not-pale persons | are | persons who look poetical".
8. "Some | contradicted persons | are | persons who keep their temper".
9. "All | 'I's' | are | persons who do not neglect important business".
10. "All | difficult things | are | things that need attention".
11. "All | unwholesome things | are | things that should be avoided".
12. "All | laws passed last week | are | laws relating to excise".
13. "All | logical studies | are | things that puzzle me".
14. "No | persons in the house | are | Jews".
15. "Some | not well-cooked dishes | are | unwholesome dishes".
16. "All | unexciting books | are | books that make one drowsy".
17. "All | men who know what they're about | are | men who can detect a sharper".
18. "All | Members of the Class 'you and I' | are | persons who know what they're about".
19. "Some | bald persons | are | persons accustomed to wear wigs".
20. "All | fully occupied persons | are | persons who do not talk about their grievances".
21. "No | riddles that can be solved | are | riddles that interest me".

Answers to § 2.



Answers to § 3.

- | | |
|--|--|
| <ol style="list-style-type: none"> 1. Some xy exist, or some x are y,
or some y are x. 2. No information. 3. All y' are x'. 4. No xy exist, &c. 5. All y' are x. 6. All x' are y. 7. All x are y. 8. All x' are y', and all y are x. 9. All x' are y'. 10. All x are y'. | <ol style="list-style-type: none"> 11. No information. 12. Some $x'y'$ exist, &c. 13. Some xy' exist, &c. 14. No xy' exist, &c. 15. Some xy exist, &c. 16. All y are x. 17. All x' are y, and all y' are x. 18. All x are y' and all y are x'. 19. All x are y, and all y' are x'. 20. All y are x'. |
|--|--|

Answers to § 4.

1. No x' are y' .
2. Some x' are y' .
3. Some x are y' .
4. [No Concl. Fallacy of Like Eliminands not asserted to exist.]
5. Some x' are y' .
6. [No Concl. Fallacy of Like Eliminands not asserted to exist.]
7. Some x are y' .
8. Some x' are y' .
9. [No Concl. Fallacy of Unlike Eliminands with an Entity-Premiss.]
10. All x are y , and all y' are x' .
11. [No Concl. Fallacy of Like Eliminands not asserted to exist.]
12. All y are x' .
13. No x' are y .
14. No x' are y' .
15. No x are y .
16. All x are y' , and all y are x' .

17. No x are y' .
18. No x are y .
19. Some x are y' .
20. No x are y' .
21. Some y are x' .
22. [No Concl. Fallacy of Unlike Eliminands with an Entity-Premiss.]
23. Some x are y .
24. All y are x' .
25. Some y are x' .
26. All y are x .
27. All x are y , and all y' are x' .
28. Some y are x' .
29. [No Concl. Fallacy of Like Eliminands not asserted to exist.]
30. Some y are x' .
31. [No Concl. Fallacy of Like Eliminands not asserted to exist.]
32. No x are y' .
33. [No Concl. Fallacy of Like Eliminands not asserted to exist.]
34. Some x are y .
35. All y are x' .
36. Some y are x' .
37. Some x are y' .
38. No y' are x .
39. Some x' are y' .
40. All y' are x' .
41. All x are y' .
42. No x are y .

Answers to § 5.

1. Somebody who has been out for a walk is feeling better.
 2. No one but John knows what the letter is about.
 3. You and I like walking.
 4. Honesty is sometimes the best policy.
 5. Some greyhounds are not fat.
 6. Some brave persons get their deserts.
 7. Some rich persons are not Esquimaux.
 8. [No Concl. Fallacy of Unlike Eliminands with an Entity-Premiss.]
 9. John is ill.
 10. Some things, that are not umbrellas, should be left behind on a journey.
 11. No music is worth paying for, unless it causes vibration in the air
 12. Some holidays are tiresome.
 13. Englishmen are not Frenchmen.
 14. No photograph of a lady is satisfactory.
 15. No one looks poetical unless he is phlegmatic.
 16. Some thin persons are not cheerful.
 17. Some persons, when contradicted, do not exercise self-control.
 18. Pigs are not fed on barley-water.
 19. Some black rabbits are not old.
- Ex. 100—102 ; Sol. 145—147.]

20. [No Concl. Fallacy of Unlike Eliminands with an Entity-Premiss.]
21. [No Concl. Fallacy of Like Eliminands not asserted to exist.]
22. Some lessons need attention.
23. [No Concl. Fallacy of Like Eliminands not asserted to exist.]
24. No one, who forgets a promise, fails to do mischief.
25. Some greedy creatures cannot fly.
26. [No Concl. Fallacy of Unlike Eliminands with an Entity-Premiss.]
27. No bride-cakes are things that need not be avoided.
28. John is happy.
29. Some people, who are not gamblers, are not philosophers
30. [No Concl. Fallacy of Unlike Eliminands with an Entity-Premiss.]
31. None of my lodgers write poetry.
32. Senna is not nice.
33. [No Concl. Fallacy of Unlike Eliminands with an Entity-Premiss.]
34. [No Concl. Fallacy of Like Eliminands not asserted to exist.]
35. Logic is unintelligible.
36. Some wild creatures are fat.
37. All wasps are unwelcome.
38. All black rabbits are young.
39. Some hard-boiled things can be cracked.
40. No antelopes fail to delight the eye.
41. All well-fed canaries are cheerful.
42. Some poetry is not producible at will.
43. No country infested by dragons fails to be fascinating.
44. [No Concl. Fallacy of Like Eliminands not asserted to exist.]
45. Some picturesque things are not made of sugar.
46. No children can sit still.
47. Some cats cannot whistle.
48. You are terrible.
49. Some oysters are not amusing.
50. Nobody in the house has a beard a yard long.
51. Some ill-fed canaries are unhappy.
52. All my sisters are unable to sing.
53. [No Concl. Fallacy of Unlike Eliminands with an Entity-Premiss.]
54. Some rich things are nice.
55. My cousins are none of them judges, and judges are none of them
cousins of mine.
56. Something wearisome is not eagerly wished for.
57. Senna is nasty.
58. [No Concl. Fallacy of Unlike Eliminands with an Entity-Premiss.]
59. Niggers are not any of them tall.
60. Some obstinate persons are not philosophers.
61. John is happy.
62. Some unwholesome dishes are not present here (i.e. cannot be
spoken of as "these").
63. No books suit feverish patients unless they make one drowsy.
64. Some greedy creatures cannot fly.
65. You and I can detect a sharper.
66. Some dreams are not lambs.

67. No lizard needs a hairbrush.
68. Some things, that may escape notice, are not battles.
69. My cousins are not any of them judges.
70. Some hard-boiled things can be cracked.
71. [No Concl. Fallacy of Unlike Eliminands with an Entity-Premiss.]
72. She is unpopular.
73. Some people, who wear wigs, are not children of yours.
74. No lobsters expect impossibilities.
75. No nightmare is eagerly desired.
76. Some nice things are not plumcakes.
77. Some kinds of jam need not be shunned.
78. All ducks are ungraceful.
79. [No Concl. Fallacy of Like Eliminands not asserted to exist.]
80. No man, who begs in the street, should fail to keep accounts.
81. Some savage creatures are not spiders.
82. [No Concl. Fallacy of Unlike Eliminands with an Entity-Premiss.]
83. No travelers, who do not carry plenty of small change, fail to lose their luggage.
84. [No Concl. Fallacy of Unlike Eliminands with an Entity-Premiss.]
85. Judges are none of them cousins of mine.
86. All my lodgers are sane.
87. Those who are fully occupied are contented, and discontented people are not fully occupied.
88. None of my cousins are judges.
89. No nightingale dislikes sugar.
90. [No Concl. Fallacy of Like Eliminands not asserted to exist.]
91. Some excuses are not clear explanations.
92. [No Concl. Fallacy of Like Eliminands not asserted to exist.]
93. No kind deed need cause scruple.
94. [No Concl. Fallacy of Like Eliminands not asserted to exist.]
95. [No Concl. Fallacy of Like Eliminands not asserted to exist.]
96. No cheats are trustworthy.
97. No clever child of mine is greedy.
98. Some things, that are meant to amuse, are not Acts of Parliament.
99. No tour, that is ever forgotten, is worth writing a book about.
100. No obedient child of mine is contented.
101. Your visit does not annoy me.

Answers to § 6.

1. Conclusion right.
2. No Concl. Fallacy of Like Eliminands not asserted to exist.
- 3, 4, 5. Concl. right.
6. No Concl. Fallacy of Like Eliminands not asserted to exist.
7. No Concl. Fallacy of Unlike Eliminands with an Entity-Premiss.
- 8—15. Concl. right.

Ex. 104—106 ; Sol. 147, 148.]

16. No Concl. Fallacy of Like Eliminands not asserted to exist.
 17—21. Concl. right.
 22. Concl. wrong: the right one is "Some x are y ."
 23—27. Concl. right.
 28. No Concl. Fallacy of Like Eliminands not asserted to exist.
 29—33. Concl. right.
 34. No Concl. Fallacy of Unlike Eliminands with an Entity-Premiss.
 35, 36, 37. Concl. right.
 38. No Concl. Fallacy of Like Eliminands not asserted to exist.
 39, 40. Concl. right.

Answers to § 7.

- 1, 2, 3. Concl. right.
 4. Concl. wrong: right one is "Some epicures are not uncles of mine."
 5. Concl. wrong: right one is "Nothing light will silence him."
 6. No Concl. Fallacy of Unlike Eliminands with an Entity-Premiss.
 7. Concl. wrong: right one is "The publication, in which I saw it, tells lies."
 8. No Concl. Fallacy of Unlike Eliminands with an Entity-Premiss.
 9. Concl. wrong: right one is "Some tedious songs are not his."
 10. Concl. right.
 11. No Concl. Fallacy of Unlike Eliminands with an Entity-Premiss.
 12. Concl. wrong: right one is "Some fierce creatures do not drink coffee."
 13. No Concl. Fallacy of Unlike Eliminands with an Entity-Premiss.
 14. Concl. right.
 15. Concl. wrong: right one is "Some shallow persons are not students."
 16. No Concl. Fallacy of Like Eliminands not asserted to exist.
 17. Concl. wrong: right one is "Some business, other than railways, is unprofitable."
 18. Concl. wrong: right one is "Some vain persons are not Professors."
 19. Concl. right.
 20. Concl. wrong: right one is "Wasps are not puppies."
 21. No Concl. Fallacy of Unlike Eliminands with an Entity-Premiss.
 22. No Concl. Same Fallacy.
 23. Concl. right.
 24. Concl. wrong: right one is "Some chocolate-creams are delicious."
 25. No Concl. Fallacy of Like Eliminands not asserted to exist.
 26. No Concl. Fallacy of Unlike Eliminands with an Entity-Premiss.
 27. Concl. wrong: right one is "Some pillows are not pokers."
 28. Concl. right.
 29. No Concl. Fallacy of Unlike Eliminands with an Entity-Premiss.
 30. No Concl. Fallacy of Like Eliminands not asserted to exist.
 31. Concl. right.
 32. No Concl. Fallacy of Like Eliminands not asserted to exist.
 33. No Concl. Fallacy of Unlike Eliminands with an Entity-Premiss.

34. Concl. wrong: right one is "Some dreaded persons are not begged to prolong their visits."
 35. Concl. wrong: right one is "No man walks on neither."
 36. Concl. right.
 37. No Concl. Fallacy of Unlike Eliminands with an Entity-Premiss.
 38. Concl. wrong: right one is "Some persons, dreaded by children, are not emperors."
 39. Concl. incomplete: the omitted portion is "Sugar is not salt."
 40. Concl. right.

Answers to § 8.

- | | | | | |
|------------------------|-----------------|------------------|-----------------|-----------------|
| 1. $a_1b_0 + b_1a_0$. | 7. $a'_c{}_0$. | 13. c_1b_0 . | 19. a_1d_0 . | 25. $e_1c'_0$. |
| 2. d_1a_0 . | 8. $c_1a'_0$. | 14. $d_1e'_0$. | 20. ac_0 . | 26. $e_1c'_0$. |
| 3. ac_0 . | 9. $c'd_0$. | 15. $e_1a'_0$. | 21. de_0 . | 27. hk'_0 . |
| 4. a_1d_0 . | 10. b_1a_0 . | 16. $b'_c{}_0$. | 22. $a_1b'_0$. | 28. $e_1d'_0$. |
| 5. cd_0 . | 11. d_1b_0 . | 17. a_1b_0 . | 23. h_1c_0 . | 29. $l'a_0$. |
| 6. d_1c_0 . | 12. $a'd_0$. | 18. d_1c_0 . | 24. c_1a_0 . | 30. $k_1b'_0$. |

Answers to § 9.

1. Babies cannot manage crocodiles.
2. *Your* presents to me are not made of tin.
3. The potatoes in this dish are old ones.
4. My servants never say "shpoonj."
5. My poultry are not officers.
6. None of *your* sons are fit to serve on a jury.
7. No pencils of mine are sugar-plums.
8. Jenkins is inexperienced.
9. No comet has a curly tail.
10. No hedge-hog takes in the *Times*.
11. This dish is unwholesome.
12. Showy talkers are not really well-informed.
13. All humming-birds are small.
14. No one with a hooked nose ever fails to make money.
15. No acrobatic feat, which involves turning a quadruple somersault, is
16. My gardener is very old. [ever attempted in a circus.
17. No gray ducks in this village wear lace collars.
18. No M.P. should ride in a donkey-race, unless he has perfect self-
19. No jug in this cupboard will hold water. [command.
20. Guinea-pigs never really appreciate Beethoven.
21. Puppies, that will not lie still, never care to do worsted work.
22. These apples were grown in the sun.

23. No goods in this shop, that are still on sale, may be carried away.
 24. No scentless flowers please me.
 25. No name in this list is unmelodious.
 26. An egg of the Great Auk is not to be had for a song.
 27. No books sold here have gilt edges, unless they are priced at 5s. and upwards.
 28. When you cut your finger, you will find Tincture of Calendula useful.
 29. All the gluttons in my family are unhealthy.
 30. All the romances in this library are well-written.
 31. No bird in this aviary lives on mince-pies.
 32. No plum-pudding, that has not been boiled in a cloth, can be distinguished from soup.
 33. None but red-haired boys learn Greek in this school.
 34. Wedding-cake always disagrees with me.
 35. *I* have never come across a mermaid at sea.
 36. Any discussion, that goes on while Tomkins is in the chair, endangers the peacefulness of our Debating-Club.
 37. All *my* friends dine at the lower table.
 38. Rainbows are not worth writing odes to.
 39. No pawnbroker is dishonest.
 40. No kitten with green eyes will play with a gorilla.
 41. All *your* poems are uninteresting.
 42. None of my peaches have been grown in a hot-house.
 43. No Mandarin ever reads Hogg's poems.
 44. My writing-desk is full of live scorpions.
 45. Shakespeare was clever.
 46. These Sorites-examples are difficult.
 47. Donkeys are not easy to swallow.
 48. Opium-eaters never wear white kid gloves.
 49. All the English pictures here are painted in oils.
 50. All the animals in the yard gnaw bones.
 51. Bathing-machines are never made of mother-of-pearl.
 52. A good husband always comes home for his tea.
 53. Rainy days are always cloudy.
 54. No cheque of yours, received this year, is payable to order.
 55. I cannot read any of Brown's letters.
 56. No heavy fish is unkind to children.
 57. No engine-driver lives on barley-sugar.
 58. All my dreams come true.
 59. No badger can guess a conundrum.
 60. I always avoid a kangaroo.
-

CHAPTER III.

SOLUTIONS.

§ 1.

Propositions of Relation reduced to normal form.

Solutions for § 1.

1. The Univ. is "persons." The Individual "I" may be regarded as a Class, of persons, whose peculiar Attribute is "represented by the Name 'I'", and may be called the Class of "I's". It is evident that this Class cannot possibly contain more than one Member: hence the Sign of Quantity is "all". The verb "have been" may be replaced by the phrase "are persons who have been". The Proposition may be written thus:—

"All"	Sign of Quantity.
"I's"	Subject.
"are"	Copula.
"persons who have been out for a walk"	Predicate.

or, more briefly,

"All | I's | are | persons who have been out for a walk".

2. The Univ. and the Subject are the same as in Ex. 1. The Proposition may be written

"All | I's | are | persons who feel better".

3. Univ. is "persons". The Subject is evidently the Class of persons from which John is *excluded*: i.e. it is the Class containing all persons who are *not* "John".

The Sign of Quantity is "no".

The verb "has read" may be replaced by the phrase "are persons who have read".

The Proposition may be written

"No | persons who are not 'John' | are | persons who have read the letter".

Ex. 97 ; Ans. 124.]

4. Univ. is "persons". The Subject is evidently the Class of persons whose only two Members are "you and I".

Hence the Sign of Quantity is "no".

The Proposition may be written

"No | Members of the Class 'you and I' | are | old persons".

5. Univ. is "creatures". The verb "run well" may be replaced by the phrase "are creatures that run well".

The Proposition may be written

"No | fat creatures | are | creatures that run well".

6. Univ. is "persons". The Subject is evidently the Class of persons who are *not* brave.

The verb "deserve" may be replaced by the phrase "are deserving of".

The Proposition may be written.

"No | not-brave persons | are | persons deserving of the fair".

7. Univ. is "persons". The phrase "looks poetical" evidently belongs to the *Predicate*: and the *Subject* is the Class, of persons, whose peculiar Attribute is "*not-pale*". The Proposition may be written

"No | not-pale persons | are | persons who look poetical".

8. Univ. is "persons". The word "when" merely serves to show that the Attributes, "contradicted" and "keeping their temper", are possessed *at the same moment*. The Proposition may be written

"Some | contradicted persons | are | persons who keep their tempers".

9. Univ. is "persons". The phrase "never neglect" is merely a stronger form of the phrase "am a person who does not neglect". The Proposition may be written

"All | 'I's' | are | persons who do not neglect important business".

10. Univ. is "things". The phrase "what is difficult" (*i.e.* "that which is difficult") is equivalent to the phrase "all difficult things". The Proposition may be written

"All | difficult things | are | things that need attention".

11. Univ. is "things". The phrase "what is unwholesome" may be interpreted as in Ex. 12. The Proposition may be written

"All | unwholesome things | are | things that should be avoided".

12. Univ. is "laws". The Predicate is evidently a Class whose peculiar Attribute is "relating to excise". The Proposition may be written

"All | laws passed last week | are | laws relating to excise".

13. Univ. is "things". The Subject is evidently the Class, of studies, whose peculiar Attribute is "logical": hence the Sign of Quantity is "all". The Proposition may be written

"All | logical studies | are | things that puzzle me".

14. Univ. is "persons". The Subject is evidently "persons in the house". The Proposition may be written

"No | persons in the house | are | Jews".

15. Univ. is "dishes". The phrase "if not well-cooked" is equivalent to the Attribute "not well-cooked". The Proposition may be written

"Some | not well-cooked dishes | are | unwholesome dishes".

16. Univ. is "books". The phrase "make one drowsy" may be replaced by the phrase "are books that make one drowsy". The Sign of Quantity is evidently "all". The Proposition may be written

"All | unexciting books | are | books that make one drowsy".

17. Univ. is "men". The Subject is evidently "a man who knows what he's about"; and the word "when" shows that the Proposition is asserted of *every* such man, *i.e.* of *all* such men. The verb "can" may be replaced by "are men who can". The Proposition may be written

"All | men who know what they're about | are | men who can detect a sharper".

18. The Univ. and the Subject are the same as in Ex. 4. The Proposition may be written

"All | Members of the Class 'you and I' | are | persons who know what they're about".

19. Univ. is "persons". The verb "wear" may be replaced by the phrase "are accustomed to wear". The Proposition may be written

"Some | bald persons | are | persons accustomed to wear wigs".

20. Univ. is "persons". The phrase "never talk" is merely a stronger form of "are persons who do not talk". The Proposition may be written

"All | fully occupied persons | are | persons who do not talk about their grievances".

21. Univ. is "riddles". The phrase "if they can be solved" is equivalent to the Attribute "that can be solved". The Proposition may be written

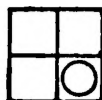
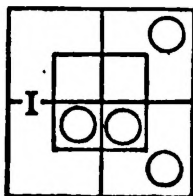
"No | riddles that can be solved | are | riddles that interest me".

§ 2.

Method of Diagrams.

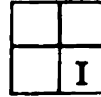
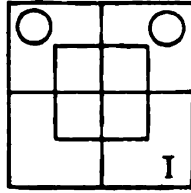
Solutions for § 4, Nos. 1—12.

1. No m are x' ;
All m' are y .



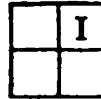
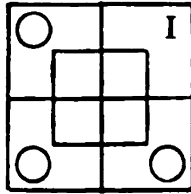
\therefore No x' are y' .

2. No m' are x ;
Some m' are y' .



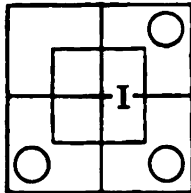
∴ Some x' are y' .

3. All m' are x ;
All m' are y' .



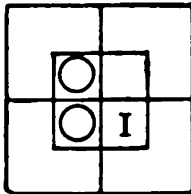
∴ Some x are y' .

4. No x' are m' ;
All y' are m .



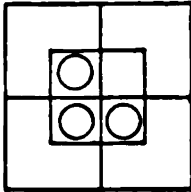
There is no Conclusion.

5. Some m are x' ;
No y are m .



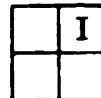
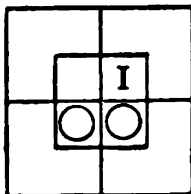
∴ Some x' are y' .

6. No x' are m ;
No m are y .



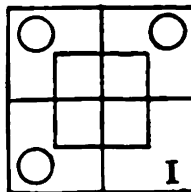
There is no Conclusion.

7. No m are x' ;
Some y' are m .



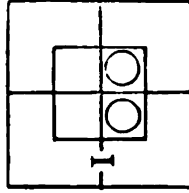
∴ Some x are y' .

8. All m' are x' ;
No m' are y .



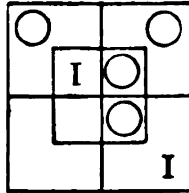
∴ Some x' are y' .

9. Some x' are m' ;
No m are y' .



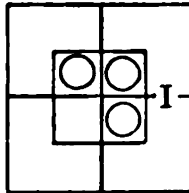
There is no Conclusion.

10. All x are m ;
All y' are m' .



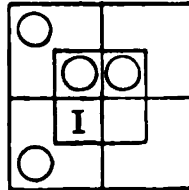
\therefore All x are y ;
All y' are x' .

11. No m are x ;
All y' are m' .



There is no Conclusion.

12. No x are m ;
All y are m .



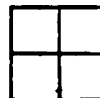
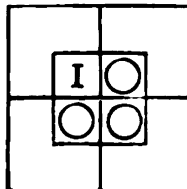
\therefore All y are x' .

Solutions for § 5, Nos. 1—12.

1. I have been out for a walk ;
I am feeling better.

Jniv. is "persons" ; m = the Class of I's ; x = who have been &c. ;
 y = who are feeling &c.

All m are x ;
All m are y .



\therefore Some x are y .

e. Somebody, who has been out for a walk, is feeling better.

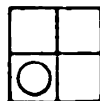
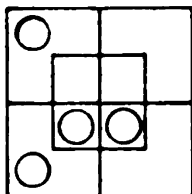
Ex. 100, 101 ; Ans. 126, 127.]

2. No one has read the letter but John ;

No one, who has *not* read it, knows what it is about.

Univ. is "persons" ; m = persons who have read &c. : x = the Class of Johns ; y = persons who know &c.

No x' are m ;
No m' are y



\therefore No x' are y

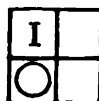
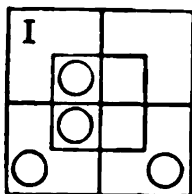
i.e. No one, but John, knows what the letter is about.

3. Those who are not old like walking ;

You and I are young.

Univ. is "persons" ; m = old ; x = persons who like &c. ; y = you and I.

All m' are x ;
All y are m' .



\therefore All y are x .

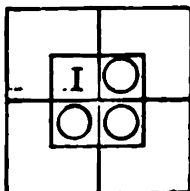
i.e. You and I like walking.

4. Your course is always honest ;

Your course is always the best policy.

Univ. is "courses" ; m = your ; x = honest ; y = courses which are the best policy.

All m are x ;
All m are y .



\therefore Some x are y .

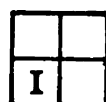
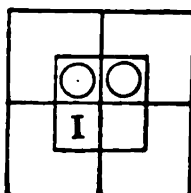
i.e. Honesty is sometimes the best policy.

5. No fat creatures run well ;

Some greyhounds run well.

Univ. is "creatures" ; m = creatures that run well : x = fat ; y = greyhounds.

No x are m ;
Some y are m .



\therefore Some y are x' .

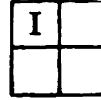
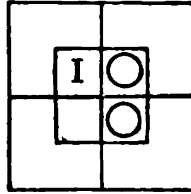
i.e. Some greyhounds are not fat.

6. Some, who deserve the fair, get their deserts ;

None but the brave deserve the fair.

Univ. is "persons" ; m = persons who deserve &c. ; x = persons who get &c. ; y = brave.

Some m are x ;
No y' are m .



\therefore Some y are x .

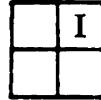
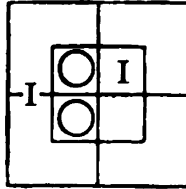
i.e. Some brave persons get their deserts.

7. Some Jews are rich ;

All Esquimaux are Gentiles.

Univ. is "persons" ; m = Jews ; x = rich ; y = Esquimaux.

Some m are x ;
All y are m' .



\therefore Some x are y' .

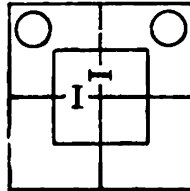
i.e. Some rich persons are not Esquimaux.

8. Sugar is sweet ;

Some sweet things are liked by children.

Univ. is "things" ; m = sweet ; x = sugar ; y = things that are liked &c ;

All x are m ;
Some m are y .



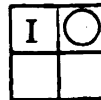
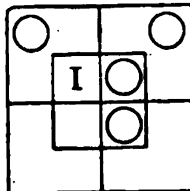
There is no Conclusion.

9. John is in the house ;

Everybody in the house is ill.

Univ. is "persons" ; m = persons in the house ; x = the Class of Johns ;
 x = ill.

All x are m ;
All m are y .



\therefore All x are y .

i.e. John is ill.

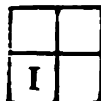
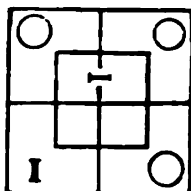
Ex. 101 ; Ans 127.]

10. Umbrellas are useful on a journey ;

What is useless on a journey should be left behind.

Univ. is "things" ; m = useful on &c. ; x = umbrellas ; y = things that should be &c.

All x are m ;
All m' are y .



\therefore Some x' are y .

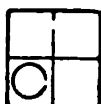
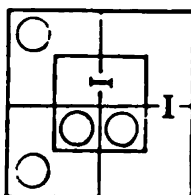
i.e. Some things that are not umbrellas, should be left behind on a journey.

11. Audible music causes vibration in the air ;

Inaudible music is not worth paying for.

Univ. is "music" ; m = audible ; x = music that causes &c. ; y = worth paying for.

All m are x ;
All m' are y' .



\therefore No x' are y .

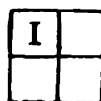
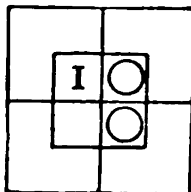
i.e. No music is worth paying for, unless it causes vibration in the air.

12. Some holidays are rainy ;

Rainy days are tiresome.

Univ. is "days" ; m = rainy ; x = holidays ; y = tiresome.

Some x are m ;
All m are y .



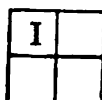
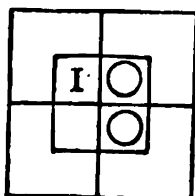
\therefore Some x are y .

i.e. Some holidays are tiresome.

Solutions for § 6, Nos. 1—10.

1.

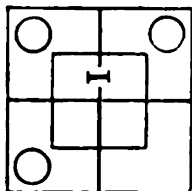
Some x are m ; No m are y' . Some x are y .



Hence proposed Conclusion is right.

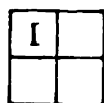
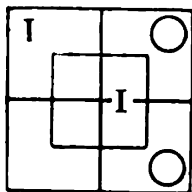
[Ex. 101, 106 ; Ans. 127, 129.]

2.

All x are m ; No y are m' . No y are x' .

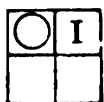
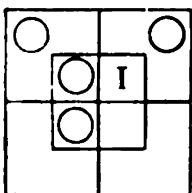
There is no Conclusion.

3.

Some x are m' ; All y' are m . Some x are y .

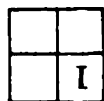
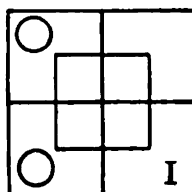
Hence proposed Conclusion is right.

4.

All x are m ; No y are m . All x are y' .

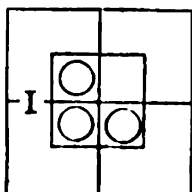
Hence proposed Conclusion is right.

5.

Some m' are x' ; No m' are y . Some x' are y' .

Hence proposed Conclusion is right.

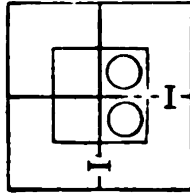
6.

No x' are m ; All y are m' . All y are x .

There is no Conclusion.

7.

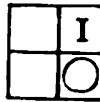
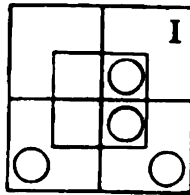
Some m' are x' ; All y' are m' . Some x' are y' .



There is no Conclusion.

8.

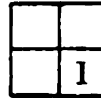
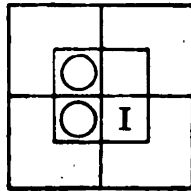
No m' are x' ; All y' are m' . All y' are x .



Hence proposed Conclusion is right.

9.

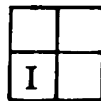
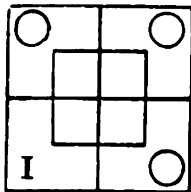
Some m are x' ; No m are y . Some x' are y' .



Hence proposed Conclusion is right.

10.

All m' are x' ; All m' are y . Some y are x' .



Hence proposed Conclusion is right.

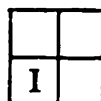
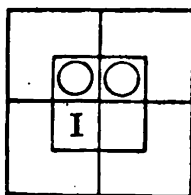
Solutions for § 7, Nos. 1—6.

1.

No doctors are enthusiastic ;

You are enthusiastic.

You are not a doctor.

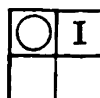
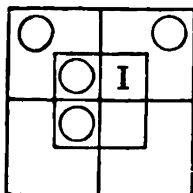
Univ. "persons" ; m = enthusiastic ; x = doctors ; y = you.No x are m ;
All y are m . \therefore Some y are x' .i.e. (since y is an *Individual*) All y are x' .
Hence proposed Conclusion is right.

2.

All dictionaries are useful ;

No useful books should be taken as fuel.

Dictionaries should not be taken as fuel.

Univ. "books" ; m = useful ; x = dictionaries ; y = may be taken &c.All x are m ;
No m are y . \therefore All x are y' .

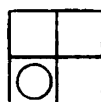
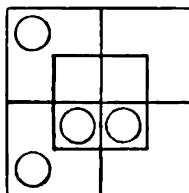
Hence proposed Conclusion is right.

3.

No misers are unselfish ;

None but misers save egg-shells.

No unselfish people save egg-shells.

Univ. "persons" ; m = misers ; x = selfish ; y = saving &c.No m are x' ;
No m' are y . \therefore No x' are y .

Hence proposed Conclusion is right.

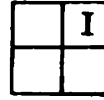
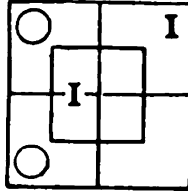
Ex. 107 ; Ans. 129.]

4.

Some epicures are ungenerous ;

All my uncles are generous.

My uncles are not epicures.

Univ. "persons" ; m = generous ; x = epicures ; y = my uncles.Some x are m' ;
All y are m . \therefore Some x are y' .

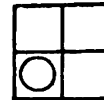
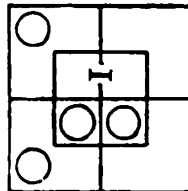
Hence proposed Conclusion is wrong, the right one being "Some epicure are not uncles of mine."

5.

Gold is heavy ;

Nothing but gold will silence him.

Nothing light will silence him.

Univ. "things" ; m = gold ; x = heavy ; y = able to silence him.All m are x ;
No m' are y . \therefore No x' are y .

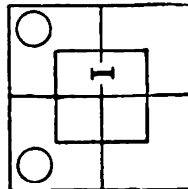
Hence proposed Conclusion is right.

6.

Some healthy people are fat ;

No unhealthy people are strong.

Some fat people are not strong.

Univ. "persons" ; m = healthy ; x = fat ; y = strong.Some m are x ;
No m' are y .

There is no Conclusion.

§ 3.

*Method of Subscripts.**Solutions for § 4.*

1. $mx'_0 \dagger m'_1y'_0 \mathbb{P} xy'_0$ [Fig. I.
i.e. "No x are y ."
2. $m'_0x_0 \dagger m'y'_1 \mathbb{P} xy'_1$ [Fig. II.
i.e. "Some x are y ."
3. $m'_1x'_0 \dagger m'_1y_0 \mathbb{P} xy'_1$ [Fig. III.
i.e. "Some x are y ."
4. $x'm'_0 \dagger y'_1m'_0 \mathbb{P}$ nothing.
[Fallacy of Like Eliminands
not asserted to exist.]
5. $mx'_1 \dagger ym_0 \mathbb{P} xy'_1$ [Fig. II.
i.e. "Some x are y ."
6. $x'm_0 \dagger my_0 \mathbb{P}$ nothing.
[Fallacy of Like Eliminands
not asserted to exist.]
7. $mx'_0 \dagger y'm_1 \mathbb{P} xy'_1$ [Fig. II.
i.e. "Some x are y ."
8. $m'_1x_0 \dagger m'y_0 \mathbb{P} xy'_1$ [Fig. III.
i.e. "Some x are y ."
9. $x'm'_1 \dagger my_0 \mathbb{P}$ nothing.
[Fallacy of Unlike Eliminands
with an Entity-Premiss.]
10. $x_1m'_0 \dagger y'_1m_0 \mathbb{P} x_1y'_0 \dagger y'_1x_0$
[Fig. I. (β).
i.e. "All x are y , and all y are x ."
11. $mx_0 \dagger y'_1m_0 \mathbb{P}$ nothing.
[Fallacy of Like Eliminands
not asserted to exist.]
12. $xm_0 \dagger y_1m'_0 \mathbb{P} y_1x_0$ [Fig. I. (α).
i.e. "All y are x ."
13. $m'_1x'_0 \dagger ym_0 \mathbb{P} xy'_0$ [Fig. I.
i.e. "No x are y ."
14. $m_1x'_0 \dagger m'_1y'_0 \mathbb{P} xy'_0$ [Fig. I.
i.e. "No x are y ."
15. $xm_0 \dagger m'y_0 \mathbb{P} xy_0$ [Fig. I.
i.e. "No x are y ."
16. $x_1m_0 \dagger y_1m'_0 \mathbb{P} (x_1y_0 \dagger y_1x_0)$
[Fig. I. (β)
i.e. "All x are y and all y are x ."
17. $xm_0 \dagger m'_1y'_0 \mathbb{P} xy'_0$ [Fig. I.
i.e. "No x are y ."
18. $xm'_0 \dagger my_0 \mathbb{P} xy_0$ [Fig. I.
i.e. "No x are y ."
19. $m_1x'_0 \dagger m_1y_0 \mathbb{P} xy'_1$ [Fig. III.
i.e. "Some x are y ."
20. $mx_0 \dagger m'_1y'_0 \mathbb{P} xy'_0$ [Fig. I.
i.e. "No x are y ."
21. $x_1m'_0 \dagger m'y_1 \mathbb{P} xy'_1$ Fig. II.
i.e. "Some x are y ."
22. $xm_1 \dagger y_1m'_0 \mathbb{P}$ nothing.
[Fallacy of Unlike Eliminands
with an Entity-Premiss.]
23. $m_1x'_0 \dagger ym_1 \mathbb{P} xy_1$ [Fig. II.
i.e. "Some x are y ."
24. $xm_0 \dagger y_1m'_0 \mathbb{P} y_1x_0$ [Fig. I. (α).
i.e. "All y are x ."
25. $mx'_1 \dagger my'_0 \mathbb{P} xy'_1$ Fig. II.
i.e. "Some x are y ."
26. $mx'_0 \dagger y_1m'_0 \mathbb{P} y_1x'_0$ [Fig. I. (α)
i.e. "All y are x ."
27. $x_1m_0 \dagger y'_1m'_0 \mathbb{P} (x_1y'_0 \dagger y'_1x_0)$
[Fig. I. (β)
i.e. "All x are y , and all y are x ."
28. $m_1x_0 \dagger my_1 \mathbb{P} xy_1$ [Fig. II.
i.e. "Some x are y ."
29. $mx_0 \dagger y_1m_0 \mathbb{P}$ nothing.
[Fallacy of Like Eliminands
not asserted to exist.]
30. $x_1m'_0 \dagger ym'_1 \mathbb{P} xy'_1$ Fig. II.
i.e. "Some y are x ."
31. $x_1m'_0 \dagger y_1m'_0 \mathbb{P}$ nothing.
[Fallacy of Like Eliminands
not asserted to exist.]

32. $xm'_0 \dagger m_1y'_0 \mathbb{P} xy'_0$ [Fig. I.
i.e. "No x are y ."

33. $mx_0 \dagger my_0 \mathbb{P}$ nothing.
[Fallacy of Like Eliminands
not asserted to exist.]

34. $mx'_0 \dagger ym_1 \mathbb{P} xy_1$ [Fig. II.
i.e. "Some x are y ."

35. $mx_0 \dagger y_1m'_0 \mathbb{P} y_1x_0$ [Fig. I. (a).
i.e. "All y are x ."

36. $m_1x_0 \dagger ym_1 \mathbb{P} x'y_1$ [Fig. II.
i.e. "Some x ' are y ."

37. $m_1x'_0 \dagger ym_0 \mathbb{P} xy'_1$ [Fig. III.
i.e. "Some x are y ."

38. $mx_0 \dagger m'y_0 \mathbb{P} xy_0$ [Fig. I.
i.e. "No x are y ."

39. $mx'_1 \dagger my_0 \mathbb{P} x'y'_1$ [Fig. II.
i.e. "Some x ' are y ."

40. $x'm_0 \dagger y'_1m'_0 \mathbb{P} y'_1x'_0$ [Fig. I. (a).
i.e. "All y ' are x ."

41. $x_1m_0 \dagger ym'_0 \mathbb{P} x_1y_0$ [Fig. I. (a).
i.e. "All x are y ."

42. $m'_x0 \dagger ym_0 \mathbb{P} xy_0$ [Fig. I.
i.e. "No x are y ."

Solutions for § 5, Nos. 13—24.

13. No Frenchmen like plumpudding ;

All Englishmen like plumpudding.

Univ. "men" ; m = liking plumpudding ; x = French ; y = English.

$xm_0 \dagger y_1m'_0 \mathbb{P} y_1x_0$ [Fig. I(a)]

i.e. Englishmen are not Frenchmen.

14. No portrait of a lady, that makes her simper or scowl, is satisfactory ;

No photograph of a lady ever fails to make her simper or scowl.

Univ. "portraits of ladies" ; m = making the subject simper or scowl ;
 x = photographic ; y = satisfactory.

$my_0 \dagger xm'_0 \mathbb{P} xy_0$ [Fig. I.]

i.e. No photograph of a lady is satisfactory.

15. All pale people are phlegmatic ;

No one looks poetical unless he is pale.

Univ. "people" ; m = pale ; x = phlegmatic ; y = looking poetical.

$m_1x'_0 \dagger m'y_0 \mathbb{P} x'y_0$ [Fig. I.]

i.e. No one looks poetical unless he is phlegmatic.

16. No old misers are cheerful ;

Some old misers are thin.

Univ. "persons" ; m = old misers ; x = cheerful ; y = thin.

$mx_0 \dagger my_1 \mathbb{P} x'y_1$ [Fig. II.]

i.e. Some thin persons are not cheerful.

17. No one, when exercising self-control, fails to keep his temper ;

Some persons, when contradicted, lose their tempers.

Univ. "persons" ; m = keeping their tempers : x = exercising self-control ;
 y = being contradicted.

$xm'_0 \dagger ym'_1 \mathbb{P} x'y_1$ [Fig. II.]

i.e. Some persons, when contradicted, do not exercise self-control.

[Ex. 100, 101 ; Ans. 127.]

18. All pigs are fat ;

Nothing that is fed on barley-water is fat.

Univ. is "things" ; m = fat ; x = pigs ; y = fed on, &c.

$$x_1 m'_0 \uparrow y m_0 \not\vdash x_1 y_0 \quad [\text{Fig. I. (a).}]$$

i.e. Pigs are not fed on barley-water.

19. All rabbits, that are not greedy, are black ;

No old rabbits are free from greediness.

Univ. is "rabbits" ; m = greedy ; x = black ; y = old.

$$m'_1 x'_0 \uparrow y m'_0 \not\vdash x y'_1 \quad [\text{Fig. III.}]$$

i.e. Some black rabbits are not old.

20. Some pictures are not first attempts ;

No first attempts are really good.

Univ. is "things" ; m = first attempts ; x = pictures ; y = really good.

$$x m'_1 \uparrow y m_0 \not\vdash \text{nothing.}$$

[Fallacy of Unlike Eliminands with an Entity-Premiss]

21. I never neglect important business ;

Your business is unimportant.

Univ. is "business" ; m = important ; x = neglected by me ; y = your.

$$m x_0 \uparrow y_1 m_0 \not\vdash \text{nothing.}$$

[Fallacy of Like Eliminands not asserted to exist.]

22. Some lessons are difficult ;

What is difficult needs attention.

Univ. is "things" ; m = difficult ; x = lessons ; y = needing attention.

$$x m_1 \uparrow m_1 y'_0 \not\vdash x y_1$$

i.e. Some lessons need attention.

23. All clever people are popular ;

All obliging people are popular.

Univ. is "people" ; m = popular ; x = clever ; y = obliging.

$$x_1 m'_0 \uparrow y_1 m'_0 \not\vdash \text{nothing.}$$

[Fallacy of Like Eliminands not asserted to exist.]

24. Thoughtless people do mischief ;

No thoughtful person forgets a promise.

Univ. is "persons" ; m = thoughtful ; x = mischievous ; y = forgetful of a promise.

$$m'_1 x'_0 \uparrow m y_0 \not\vdash x' y_0$$

i.e. No one, who forgets a promise, fails to do mischief.

Solutions for § 6.

1. $x m_1 \uparrow m y'_0 \not\vdash x y_1$ [Fig. II.] Concl. right.

2. $x_1 m'_0 \uparrow y m'_0$ Fallacy of Like Eliminands not asserted to exist.

3. $x m'_1 \uparrow y'_1 m'_0 \not\vdash x y_1$ [Fig. II.] Concl. right.

Ex. 102, 106 ; Ans. 127—129.]

4. $x_1m'_0 \vdash ym_0 \vdash x_1y_0$ [Fig. I (a).] Concl. right.
5. $m'_x \vdash m'_y \vdash x'y_1$ [Fig. II.] „
6. $x'm_0 \vdash y_1m_0$ Fallacy of Like Eliminands not asserted to exist.
7. $m'_x \vdash y'_1m_0$ Fallacy of Unlike Eliminands with an Entity-Premiss.
8. $m'_x \vdash y'_1m_0 \vdash y'_1x_0$ [Fig. I (a).] Concl. right.
9. $mx'_1 \vdash my_0 \vdash x'y_1$ [Fig. II.] „
10. $m'_1x_0 \vdash m'_1y'_0 \vdash x'y_1$ [Fig. III.] „
11. $x_1m_0 \vdash ym_1 \vdash x'y_1$ [Fig. II.] „
12. $xm_0 \vdash m'y'_0 \vdash xy'_0$ [Fig. I.] „
13. $xm_0 \vdash y'_1m'_0 \vdash y'_1x_0$ [Fig. I (a).] „
14. $m'_1x_0 \vdash m'_1y'_0 \vdash x'y_1$ [Fig. III.] „
15. $mx'_1 \vdash y_1m_0 \vdash x'y_1$ [Fig. II.] „
16. $x'm_0 \vdash y'_1m_0$ Fallacy of Like Eliminands not asserted to exist.
17. $m'_x \vdash m'_1y_0 \vdash x'y_1$ [Fig. III.] Concl. right.
18. $x'm_0 \vdash my_1 \vdash xy_1$ [Fig. II.] „
19. $mx'_1 \vdash m_1y'_0 \vdash x'y_1$ [„] „
20. $x'm'_0 \vdash m'y'_1 \vdash xy'_1$ [„] „
21. $mx_0 \vdash m_1y_0 \vdash x'y'_1$ [Fig. III.] „
22. $x'_1m'_0 \vdash ym'_1 \vdash xy_1$ [Fig. II.] Concl. wrong: the right one is
“Some x are y .”
23. $m_1x'_0 \vdash m'y'_0 \vdash x'y'_0$ [Fig. I.] Concl. right.
24. $x_1m_0 \vdash m'_1y'_0 \vdash x_1y'_0$ [Fig. I. (a)] „
25. $xm'_0 \vdash m_1y'_0 \vdash xy'_0$ [Fig. I.] „
26. $m_1x_0 \vdash y_1m'_0 \vdash y_1x_0$ [Fig. I (a).] „
27. $x_1m'_0 \vdash my'_0 \vdash x_1y'_0$ [„] „
28. $x_1m'_0 \vdash y'm'_0$ Fallacy of Like Eliminands not asserted to exist
29. $x'm_0 \vdash m'y'_0 \vdash x'y'_0$ [Fig. I.] Concl. right.
30. $x_1m'_0 \vdash m_1y_0 \vdash x_1y_0$ [Fig. I (a).] „
31. $x'_1m_0 \vdash y'm'_0 \vdash x'_1y'_0$ [„] „
32. $xm_0 \vdash y'm'_0 \vdash xy'_0$ [Fig. I.] „
33. $m_1x_0 \vdash y'_1m'_0 \vdash y'_1x_0$ [Fig. I. (a)] „
34. $x_1m_0 \vdash ym'_1$ Fallacy of Unlike Eliminands with an Entity-Premiss.
35. $xm_1 \vdash m_1y'_0 \vdash xy_1$ [Fig. II.] Concl. right.
36. $m_1x_0 \vdash y_1m'_0 \vdash y_1x_0$ [Fig. I (a).] „
37. $mx'_0 \vdash m_1y_0 \vdash xy'_1$ [Fig. III.] „
38. $xm_0 \vdash my'_0$ Fallacy of Like Eliminands not asserted to exist.
39. $mx_0 \vdash my'_1 \vdash x'y'_1$ [Fig. II.] Concl. right.
40. $mx'_0 \vdash ym_1 \vdash xy_1$ [Fig. II.] „

Solutions for § 7.

1. No doctors are enthusiastic ;
You are enthusiastic.

You are not a doctor.

Univ. "persons" ; m = enthusiastic ; x = doctors ; y = you.

$$xm_0 \dagger y_1m'_0 \mathbb{P} y_1x_0 \quad [\text{Fig. I}(\alpha).]$$

Conclusion right.

2. All dictionaries are useful ;
No useful books should be taken as fuel.

Dictionaries should not be taken as fuel.

Univ. "books" ; m = useful ; x = dictionaries ; y = books that may be taken as fuel.

$$x_1m'_0 \dagger my_0 \mathbb{P} x_1y_0 \quad [\text{Fig. I}(\alpha).]$$

Conclusion right.

3. No misers are unselfish ;
None but misers save egg-shells.

No unselfish people save egg-shells.

Univ. "people" ; m = misers ; x = selfish ; y = people who save egg-shells.

$$mx'_0 \dagger m'y_0 \mathbb{P} x'y_0 \quad [\text{Fig. I.}]$$

Conclusion right.

4. Some epicures are ungenerous ;
All my uncles are generous.
My uncles are not epicures.

Univ. "persons" ; m = generous ; x = epicures ; y = my uncles.

$$xm'_1 \dagger y_1m'_0 \mathbb{P} xy'_1 \quad [\text{Fig. II.}]$$

Conclusion wrong : right one is "Some epicures are not uncles of mine."

5. Gold is heavy ;
Nothing but gold will silence him.
Nothing light will silence him.

Univ. "things" ; m = gold ; x = heavy ; y = things that will silence him.

$$m_1x'_0 \dagger m'y_0 \mathbb{P} x'y_0 \quad [\text{Fig. I.}]$$

Conclusion right.

6. Some healthy people are fat ;
No unhealthy people are strong.
Some fat people are not strong.

Univ. "people" ; m = healthy ; x = fat ; y = strong.

$$mx_1 \dagger m'y_0$$

No Conclusion. [Fallacy of Unlike Eliminands with an Entity-Premiss.]

7. I saw it in a newspaper ;
All newspapers tell lies.
It was a lie.

Univ. "publications" ; m = newspapers ; x = publications in which I saw it ; y = telling lies.

$$x_1m'_0 \dagger m_1y'_0 \mathbb{P} x_1y'_0 \quad [\text{Fig. I}(\alpha).]$$

Conclusion wrong : right one is "The publication, in which I saw it, tells lies."

Ex. 107, 108 ; Ans. 130.]

8. Some cravats are not artistic ;
I admire anything artistic.

There are some cravats that I do not admire.

Univ. "things" ; m = artistic ; x = cravats ; y = things that I admire.

$$xm'_1 \dagger m_1y'_0$$

No Conclusion. [Fallacy of Unlike Eliminands with an Entity-Premiss.]

9. His songs never last an hour.

A song, that lasts an hour, is tedious.

His songs are never tedious.

Univ. "songs" ; m = lasting an hour ; x = his ; y = tedious.

$$x_1m_0 \dagger m_1y'_0 \text{ P } x'y_1 \text{ [Fig. III.]}$$

Conclusion wrong : right one is "Some tedious songs are not his."

10. Some candles give very little light ;

Candles are *meant* to give light.

Some things, that are meant to give light, give very little.

Univ. "things" ; m = candles ; x = giving &c. ; y = meant &c.

$$mx_1 \dagger m_1y'_0 \text{ P } xy_1 \text{ [Fig. II.]}$$

Conclusion right.

11. All, who are anxious to learn, work hard.

Some of these boys work hard.

Some of these boys are anxious to learn.

Univ. "persons" ; m = hard-working ; x = anxious to learn ; y = these boys.

$$x_1m'_0 \dagger ym_1$$

No Conclusion. [Fallacy of Unlike Eliminands with an Entity-Premiss.]

12. All lions are fierce ;

Some lions do not drink coffee.

Some creatures that drink coffee are not fierce.

Univ. "creatures" ; m = lions ; x = fierce ; y = creatures that drink coffee.

$$m_1x'_0 \dagger my'_1 \text{ P } xy'_1 \text{ [Fig. II.]}$$

Conclusion wrong : right one is "Some fierce creatures do not drink coffee."

13. No misers are generous ;

Some old men are ungenerous.

Some old men are misers

Univ. "persons" ; m = generous ; x = misers ; y = old men.

$$xm_0 \dagger ym'_1$$

No Conclusion. [Fallacy of Unlike Eliminands with an Entity-Premiss.]

14. No fossil can be crossed in love ;

An oyster may be crossed in love.

Oysters are not fossils.

Univ. "things" ; m = things that can be crossed in love ; x = fossils ; y = oysters.

$$xm_0 \dagger y_1m'_0 \text{ P } y_1x_0 \text{ [Fig. I(a).]}$$

Conclusion right.

15. All uneducated people are shallow ;
Students are all educated.

No students are shallow.

Univ. "people" ; m = educated ; x = shallow ; y = students.

$$m'_1x'_0 \uparrow y_1m'_0 \text{ } \mathbb{P} \text{ } xy'_1 \text{ [Fig. III.]}$$

Conclusion wrong : right one is "Some shallow people are not students."

16. All young lambs jump ;

No young animals are healthy, unless they jump.

All young lambs are healthy.

Univ. "young animals" ; m = young animals that jump ; x = lambs ;
 y = healthy.

$$x_1m'_0 \uparrow m'y_0$$

No Conclusion. [Fallacy of Like Eliminands not asserted to exist.]

17. Ill-managed business is unprofitable ;

Railways are never ill-managed.

All railways are profitable.

Univ. "business" ; m = ill-managed ; x = profitable ; y = railways.

$$m_1x_0 \uparrow y_1m_0 \text{ } \mathbb{P} \text{ } x'y'_1 \text{ [Fig. III.]}$$

Conclusion wrong : right one is "Some business, other than railways, is unprofitable."

18. No Professors are ignorant ;

All ignorant people are vain.

No Professors are vain.

Univ. "people" ; m = ignorant ; x = Professors ; y = vain.

$$xm_0 \uparrow m_1y'_0 \text{ } \mathbb{P} \text{ } x'y_1 \text{ [Fig. III.]}$$

Conclusion wrong : right one is "Some vain persons are not Professors."

19. A prudent man shuns hyænas ;

No banker is imprudent.

No banker fails to shun hyænas.

Univ. "men" ; m = prudent ; x = shunning hyænas ; y = bankers.

$$m_1x'_0 \uparrow ym'_0 \text{ } \mathbb{P} \text{ } x'y_0 \text{ [Fig. 1.]}$$

Conclusion right.

20. All wasps are unfriendly ;

No puppies are unfriendly.

No puppies are wasps.

Univ. "creatures" ; m = friendly ; x = wasps ; y = puppies.

$$x_1m_0 \uparrow ym'_0 \text{ } \mathbb{P} \text{ } x_1y_0 \text{ [Fig. I(a).]}$$

Conclusion incomplete : complete one is "Wasps are not puppies".

21. No Jews are honest ;

Some Gentiles are rich.

Some rich people are dishonest.

Univ. "persons" ; m = Jews ; x = honest ; y = rich.

$$mx_0 \uparrow m'y_1$$

No Conclusion. [Fallacy of Unlike Eliminands with an Entity-Premiss.]

Ex. 108, 109 ; Ans. 130.]

22. No idlers win fame ;
Some painters are not idle.
Some painters win fame.

Univ. "persons"; m = idlers; x = persons who win fame; y = painters.

No Conclusion. [Fallacy of Unlike Eliminands with an Entity-Premiss.]
 $mx_0 \uparrow ym'_1$

23. No monkeys are soldiers ;
All monkeys are mischievous.
Some mischievous creatures are not soldiers.

Univ. "creatures"; m = monkeys; x = soldiers; y = mischievous.

Conclusion right.
 $mx_0 \uparrow m_1y'_0 \text{ } \text{ } x'y_1$ [Fig. III.]

24. All these bonbons are chocolate-creams ;
All these bonbons are delicious.
Chocolate-creams are delicious.

Univ. "food"; m = these bonbons; x = chocolate-creams; y = delicious.

Conclusion wrong, being in excess of the right one, which is "Some chocolate-creams are delicious."
 $m_1x'_0 \uparrow m_1y'_0 \text{ } \text{ } xy_1$ [Fig. III.]

25. No muffins are wholesome ;
All buns are unwholesome.
Buns are not muffins.

Univ. "food"; m = wholesome; x = muffins; y = buns.

No Conclusion. [Fallacy of Like Eliminands not asserted to exist.]
 $xm_0 \uparrow y_1m_0$

26. Some unauthorised reports are false ;
All authorised reports are trustworthy.
Some false reports are not trustworthy.

Univ. "reports"; m = authorised; x = true; y = trustworthy.

No Conclusion. [Fallacy of Unlike Eliminands with an Entity-Premiss.]
 $m'x'_1 \uparrow m_1y'_0$

27. Some pillows are soft ;
No pokers are soft.
Some pokers are not pillows.

Univ. "things"; m = soft; x = pillows; y = pokers.

Conclusion wrong: right one is "Some pillows are not pokers."
 $xm_1 \uparrow ym_0 \text{ } \text{ } xy'_1$ [Fig. II.]

28. Improbable stories are not easily believed ;
None of his stories are probable.

Univ. "stories"; m = probable; x = easily believed; y = his.
Conclusion right.
 $m'_1x_0 \uparrow ym_0 \text{ } \text{ } xy_0$ [Fig. I.]

29. No thieves are honest ;
Some dishonest people are found out.
Some thieves are found out.

Univ. "people" ; m = honest ; x = thieves ; y = found out.

$$xm_0 \uparrow m'y_1$$

No Conclusion. [Fallacy of Unlike Eliminands with an Entity-Premiss.]

30. No muffins are wholesome ;
All puffy food is unwholesome.
All muffins are puffy.

Univ. is "food" ; m = wholesome ; x = muffins ; y = puffy.

$$xm_0 \uparrow y_1m_0$$

No Conclusion. [Fallacy of Like Eliminands not asserted to exist.]

31. No birds, except peacocks, are proud of their tails ;
Some birds, that are proud of their tails, cannot sing.
Some peacocks cannot sing.

Univ. "birds" ; m = proud of their tails ; x = peacocks ; y = birds that can sing.

$$x'm_0 \uparrow my'_1 \text{ } \mathbb{P} \text{ } xy'_1 \text{ [Fig. II.]}$$

Conclusion right.

32. Warmth relieves pain ;
Nothing, that does not relieve pain, is useful in toothache.
Warmth is useful in toothache.

Univ. "applications" ; m = relieving pain ; x = warmth ; y = useful in toothache.

$$x_1m'_0 \uparrow m'y_0$$

No Conclusion. [Fallacy of Like Eliminands not asserted to exist.]

33. No bankrupts are rich ;
Some merchants are not bankrupts.
Some merchants are rich.

Univ. "persons" ; m = bankrupts ; x = rich ; y = merchants.

$$mx_0 \uparrow ym'_1$$

No Conclusion. [Fallacy of Unlike Eliminands with an Entity-Premiss.]

34. Bores are dreaded ;
No bore is ever begged to prolong his visit.

No one, who is dreaded, is ever begged to prolong his visit.

Univ. "persons" ; m = bores ; x = dreaded ; y = begged to &c.

$$m_1x'_0 \uparrow my_0 \text{ } \mathbb{P} \text{ } xy'_1 \text{ [Fig. III.]}$$

Conclusion wrong : the right one is "Some dreaded persons are not begged to prolong their visits."

35. All wise men walk on their feet ;
All unwise men walk on their hands.
No man walks on both.

Univ. "men" ; m = wise ; x = walking on their feet ; y = walking on their hands.

$$m_1x'_0 \uparrow m'_1y'_0 \text{ } \mathbb{P} \text{ } x'y'_0 \text{ [Fig. I.]}$$

Conclusion wrong : right one is "No man walks on neither."

Ex. 109, 110 ; Ans. 130, 131.]

36. No wheelbarrows are comfortable ;
 No uncomfortable vehicles are popular.
 No wheelbarrows are popular.

Univ. "vehicles" ; m = comfortable ; x = wheelbarrows ; y = popular.
 $xm_0 \uparrow m'x_0 \text{ P } xy_0$ [Fig. I.]

Conclusion right.

37. No frogs are poetical ;
 Some ducks are unpoetical.
 Some ducks are not frogs.

Univ. "creatures" ; m = poetical ; x = frogs ; y = ducks.
 $xm_0 \uparrow ym'_1$

No Conclusion. [Fallacy of Unlike Eliminands with an Entity-Premiss.]

38. No emperors are dentists ;
 All dentists are dreaded by children.
 No emperors are dreaded by children.

Univ. "persons" ; m = dentists ; x = emperors ; y = dreaded by children.
 $xm_0 \uparrow m_1y'_0 \text{ P } x'y_1$ [Fig. III.]

Conclusion wrong : right one is "Some persons, dreaded by children, are not emperors."

39. Sugar is sweet ;
 Salt is not sweet.
 Salt is not sugar.

Univ. "things" ; m = sweet ; x = sugar ; y = salt.
 $x_1m'_0 \uparrow y_1m_0 \text{ P } (x_1y_0 \uparrow y_1x_0)$ [Fig. I (β).]

Conclusion incomplete : omitted portion is "Sugar is not salt."

40. Every eagle can fly ;
 Some pigs cannot fly.
 Some pigs are not eagles.

Univ. "creatures" ; m = creatures that can fly ; x = eagles ; y = pigs.
 $x_1m'_0 \uparrow ym'_1 \text{ P } x'y_1$ [Fig. II.]

Conclusion right.

Solutions for § 8.

$$1. \quad \overset{1}{c}d_0 \uparrow \overset{2}{a_1}d'_0 \uparrow \overset{3}{b_1}c'_0 ; \quad \overset{1}{c}d \uparrow \overset{2}{a}d' \uparrow \overset{3}{b}c' \text{ P } ab_0 \uparrow a_1 \uparrow b_1 \text{ i.e. P } a_1b_0 \uparrow b_1a_0$$

$$2. \quad \overset{1}{d_1}b'_0 \uparrow \overset{2}{a}c'_0 \uparrow \overset{3}{b}c_0 ; \quad \overset{1}{d}b' \uparrow \overset{3}{b}c \uparrow \overset{2}{a}c' \text{ P } da_0 \uparrow d_1 \text{ i.e. P } d_1a_0$$

$$3. \quad \overset{1}{b}a_0 \uparrow \overset{2}{c}d'_0 \uparrow \overset{3}{d_1}b'_0 ; \quad \overset{1}{b}a \uparrow \overset{3}{d}b' \uparrow \overset{2}{c}d' \text{ P } ac_0$$

$$4. \quad \overset{1}{b}c_0 \uparrow \overset{2}{a_1}b'_0 \uparrow \overset{3}{c}d'_0 ; \quad \overset{1}{b}c \uparrow \overset{2}{a}b' \uparrow \overset{3}{c}d \text{ P } ad_0 \uparrow a_1 \text{ i.e. P } a_1d_0$$

(Ex. 110 ; Ans. 131.)

5. $b_1^1 a_0 + b c_0 + a' d_0$; $\underline{b}^1 \underline{a} + \underline{b}^2 c + \underline{a'}^3 d \mathbb{P} cd_0$
6. $a_1 b_0 + b' c_0 + d_1 a'_0$; $\underline{a}^1 \underline{b} + \underline{b'}^2 c + \underline{d}^3 \underline{a'} \mathbb{P} cd_0 + d_1$ i.e. $\mathbb{P} d_1 c_0$
7. $db'_0 + b_1 a'_0 + cd'_0$; $\underline{d}^1 \underline{b'} + \underline{b}^2 \underline{a'} + c^3 \underline{d'} \mathbb{P} a' c_0$
8. $b' d_0 + a' b_0 + c_1 d'_0$; $\underline{b'}^1 \underline{d} + \underline{a'}^2 \underline{b} + c^3 \underline{d'} \mathbb{P} a' c_0 + c_1$ i.e. $\mathbb{P} c_1 a'_0$
9. $b_1^1 a'_0 + ad_0 + b_1 c'_0$; $\underline{b'}^1 \underline{a'} + \underline{a}^2 d + \underline{b}^3 \underline{c'} \mathbb{P} dc'_0$
10. $cd_0 + b_1 c'_0 + ad'_0$; $\underline{c}^1 \underline{d} + \underline{b}^2 \underline{c'} + a^3 \underline{d'} \mathbb{P} ba_0 + b_1$ i.e. $\mathbb{P} b_1 a_0$
11. $bc_0 + d_1 a'_0 + c'_1 a_0$; $\underline{b}^1 \underline{c} + \underline{c'}^3 \underline{a} + \underline{d}^2 \underline{a'} \mathbb{P} bd_0 + d_1$ i.e. $\mathbb{P} d_1 b_0$
12. $cb'_0 + c'_1 d_0 + b_1 a'_0$; $\underline{c}^1 \underline{b'} + \underline{c'}^2 d + \underline{b}^3 \underline{a'} \mathbb{P} da'_0$
13. $d_1 e'_0 + c_1 a'_0 + bd'_0 + e_1 a_0$; $\underline{d}^1 \underline{e'} + \underline{b}^3 \underline{d'} + \underline{c}^4 \underline{a} + c^2 \underline{a'} \mathbb{P} bc_0 + c_1$
i.e. $\mathbb{P} c_1 b_0$
14. $c_1 b'_0 + a_1 e'_0 + d_1 b_0 + a'_1 c'_0$; $\underline{c}^1 \underline{b'} + \underline{d}^3 \underline{b} + \underline{a'}^4 \underline{c'} + \underline{a}^2 \underline{e'} \mathbb{P} de'_0 + d_1$
i.e. $\mathbb{P} d_1 e'_0$
15. $b' d_0 + e_1 c'_0 + b_1 a'_0 + d'_1 c_0$; $\underline{b'}^1 \underline{d} + \underline{b}^3 \underline{a'} + \underline{d'}^4 \underline{c} + e^2 \underline{c'} \mathbb{P} a' e_0 + e_1$
i.e. $\mathbb{P} e_1 a'_0$
16. $a' e_0 + d_1 c_0 + a_1 b'_0 + e'_1 d'_0$; $\underline{a'}^1 \underline{e} + \underline{a}^3 \underline{b'} + \underline{e'}^4 \underline{d'} + \underline{d}^2 \underline{c} \mathbb{P} b' c_0$
17. $d_1 c'_0 + a_1 e'_0 + bd'_0 + c_1 e_0$; $\underline{d}^1 \underline{c'} + \underline{b}^3 \underline{d'} + \underline{c}^4 \underline{e} + a^2 \underline{e'} \mathbb{P} ba_0 + a_1$
i.e. $\mathbb{P} a_1 b_0$
18. $a_1 b'_0 + d_1 e'_0 + a'_1 c_0 + be$; $\underline{a}^1 \underline{b'} + \underline{a'}^3 \underline{c} + \underline{b}^4 \underline{e} + \underline{d}^2 \underline{e'} \mathbb{P} cd_0 + d_1$ i.e. $\mathbb{P} d_1 c_0$
19. $bc_0 + e_1 h'_0 + a_1 b'_0 + dh_0 + e'_1 c'_0$; $\underline{b}^1 \underline{c} + \underline{a}^3 \underline{b'} + \underline{e'}^5 \underline{c'} + \underline{e}^2 \underline{h'} + \underline{d}^4 \underline{h}$
 $\mathbb{P} ad_0 + a_1$ i.e. $\mathbb{P} a_1 d_0$
20. $dh'_0 + ce_0 + h_1 b'_0 + ad'_0 + be'_0$; $\underline{d}^1 \underline{h'} + \underline{h}^3 \underline{b'} + \underline{a}^4 \underline{d'} + \underline{b}^5 \underline{e'} + c^2 \underline{e} \mathbb{P} ac_0$
21. $b_1 a'_0 + dh_0 + ce_0 + ah'_0 + c'_1 b'_0$; $\underline{b}^1 \underline{a'} + \underline{a}^4 \underline{h'} + \underline{d}^2 \underline{h} + \underline{c'}^5 \underline{b'} + \underline{c}^3 \underline{e} \mathbb{P} de_0$
22. $e_1 d_0 + b' h'_0 + c'_1 d'_0 + a_1 e'_0 + ch$; $\underline{e}^1 \underline{d} + \underline{c'}^3 \underline{d'} + \underline{a}^4 \underline{e'} + \underline{c}^5 \underline{h} + \underline{b'}^2 \underline{h'}$
 $\mathbb{P} ab'_0 + a_1$ i.e. $\mathbb{P} a_1 b'_0$

$$23. \overset{1}{b_1} \overset{2}{a_0} + \overset{3}{d_1} \overset{4}{b_0} + \overset{5}{a_1} \overset{6}{c'_0}; \quad \overset{1}{b_1} \overset{3}{a_0} + \overset{5}{d_1} \overset{2}{b_0} + \overset{4}{c_1} \overset{6}{c'_0} \\ \text{P } b_1 c_0 + b_1 \text{ i.e. P } b_1 c_1$$

$$24. \overset{1}{k_1} \overset{2}{k_0} + \overset{3}{b_1} \overset{4}{a_0} + \overset{5}{c_1} \overset{6}{d'_0} + \overset{7}{c_1} \overset{8}{k_0} + \overset{9}{d_1} \overset{10}{k'_0} + \overset{11}{b_1} \overset{12}{c'_0}; \\ \overset{1}{k_1} \overset{4}{k_0} + \overset{5}{c_1} \overset{3}{k'_0} + \overset{6}{c_1} \overset{2}{d'_0} + \overset{7}{b_1} \overset{8}{c'_0} + \overset{9}{b_1} \overset{10}{a_0} + \overset{11}{b_1} \overset{12}{c_1} \text{ i.e. P } c_1 a_0$$

$$25. \overset{1}{a_1} \overset{2}{d'_0} + \overset{3}{k_1} \overset{4}{b'_0} + \overset{5}{c_1} \overset{6}{k'_0} + \overset{7}{a_1} \overset{8}{b_0} + \overset{9}{d_1} \overset{10}{c'_0} + \overset{11}{k_1} \overset{12}{k'_0}; \\ \overset{1}{a_1} \overset{4}{d'_0} + \overset{5}{a_1} \overset{3}{b_0} + \overset{6}{k_1} \overset{2}{b'_0} + \overset{7}{d_1} \overset{8}{c'_0} + \overset{9}{k_1} \overset{10}{k'_0} + \overset{11}{c_1} \overset{12}{k'_0} \text{ P } c'_0 + c_1 \text{ i.e. P } c_1 c'_0$$

$$26. \overset{1}{a'_1} \overset{2}{k'_0} + \overset{3}{d'_0} \overset{4}{k'_0} + \overset{5}{c_1} \overset{6}{b_0} + \overset{7}{k_1} \overset{8}{k_0} + \overset{9}{a_1} \overset{10}{c_0} + \overset{11}{b_1} \overset{12}{d'_0}; \\ \overset{1}{a'_1} \overset{4}{k'_0} + \overset{5}{k_1} \overset{3}{k_0} + \overset{6}{d'_0} \overset{2}{k'_0} + \overset{7}{a_1} \overset{8}{c'_0} + \overset{9}{b_1} \overset{10}{d'_0} + \overset{11}{c_1} \overset{12}{b_0} \text{ P } c'_0 + c_1 \text{ i.e. P } c_1 c'_0$$

$$27. \overset{1}{c_1} \overset{2}{d_0} + \overset{3}{h_1} \overset{4}{b_0} + \overset{5}{a'_1} \overset{6}{k'_0} + \overset{7}{c'_0} \overset{8}{d'_0} + \overset{9}{b_1} \overset{10}{d'_0} + \overset{11}{a_1} \overset{12}{c'_0}; \\ \overset{1}{c_1} \overset{4}{d'_0} + \overset{5}{c_1} \overset{3}{d'_0} + \overset{6}{b_1} \overset{2}{d'_0} + \overset{7}{k_1} \overset{8}{b_0} + \overset{9}{a_1} \overset{10}{c'_0} + \overset{11}{a_1} \overset{12}{k'_0} \text{ P } h k'_0$$

$$28. \overset{1}{a'_1} \overset{2}{k'_0} + \overset{3}{c_1} \overset{4}{b'_0} + \overset{5}{h_1} \overset{6}{k'_0} + \overset{7}{d'_0} \overset{8}{c_0} + \overset{9}{a_1} \overset{10}{b_0} + \overset{11}{c'_1} \overset{12}{h'_0}; \\ \overset{1}{a'_1} \overset{4}{k'_0} + \overset{5}{h_1} \overset{3}{k'_0} + \overset{6}{a_1} \overset{2}{b_0} + \overset{7}{c'_1} \overset{8}{h'_0} + \overset{9}{d'_0} \overset{10}{c_0} + \overset{11}{a_1} \overset{12}{b_0} \text{ P } e d'_0 + c_1 \text{ i.e. P } c_1 d'_0$$

$$29. \overset{1}{e k_0} + \overset{2}{b'_0} \overset{3}{m_0} + \overset{4}{a'_0} \overset{5}{k'_0} + \overset{6}{h_1} \overset{7}{c'_0} + \overset{8}{d_1} \overset{9}{k'_0} + \overset{10}{c_0} \overset{11}{b_0} + \overset{12}{a'_1} \overset{13}{l'_0} + \overset{14}{h_1} \overset{15}{m'_0}; \\ \overset{1}{e k_0} + \overset{4}{h'_0} \overset{3}{c'_0} + \overset{5}{d_1} \overset{2}{k'_0} + \overset{6}{d'_1} \overset{7}{l'_0} + \overset{8}{h_1} \overset{9}{m'_0} + \overset{10}{b'_0} \overset{11}{m_0} + \overset{12}{c_0} \overset{13}{b_0} + \overset{14}{a'_0} \overset{15}{l'_0} \text{ P } l a_0$$

$$30. \overset{1}{n_1} \overset{2}{m'_0} + \overset{3}{a'_1} \overset{4}{c'_0} + \overset{5}{c'_0} \overset{6}{l_0} + \overset{7}{k_1} \overset{8}{r_0} + \overset{9}{a_1} \overset{10}{h'_0} + \overset{11}{d_1} \overset{12}{l'_0} + \overset{13}{c_1} \overset{14}{n'_0} + \overset{15}{e_1} \overset{16}{b'_0} + \overset{17}{m_1} \overset{18}{r'_0} + \overset{19}{h_1} \overset{20}{d'_0}; \\ \overset{1}{n_1} \overset{7}{m'_0} + \overset{8}{c'_0} \overset{3}{l_0} + \overset{4}{d_1} \overset{2}{l'_0} + \overset{5}{m_1} \overset{6}{r'_0} + \overset{7}{k_1} \overset{8}{r_0} + \overset{9}{h_1} \overset{10}{d'_0} + \overset{11}{a_1} \overset{12}{h'_0} + \overset{13}{a'_0} \overset{14}{c'_0} + \overset{15}{e_1} \overset{16}{b'_0} \text{ P } k b'_0 + k_1 \\ \text{i.e. P } k_1 b'_0$$

Solutions for § 9.

1.

$$\overset{1}{b_1} \overset{2}{d_0} + \overset{3}{a_0} \overset{4}{c'_0} + \overset{5}{d'_1} \overset{6}{c'_0}; \quad \overset{1}{b_1} \overset{3}{d_0} + \overset{2}{a_0} \overset{4}{c'_0} \text{ P } b a_0 + b_1, \text{ i.e. P } b_1 a_0 \\ \text{i.e. Babies cannot manage crocodiles.}$$

2.

$$\overset{1}{a_1} \overset{2}{b'_0} + \overset{3}{d'_1} \overset{4}{c'_0} + \overset{5}{b_0} \overset{6}{c_0}; \quad \overset{1}{a_1} \overset{3}{b'_0} + \overset{2}{b_0} \overset{4}{c'_0} \text{ P } a d_0 + d_1, \text{ i.e. P } d_1 a_0 \\ \text{i.e. Your presents to me are not made of tin.}$$

3.

$d_1^1 a_0 + c_1^2 b'_0 + a'^3 b_0$; $d_1^1 \underline{a} + \underline{a'}^3 \underline{b} + c^2 \underline{b'}$ $\mathbb{P} dc_0 + c_1$, i.e. $\mathbb{P} c_1 d_0$
i.e. The potatoes in this dish are old ones.

4.

$ba_0 + b'd_0 + c_1 a'_0$; $\underline{b}^1 \underline{a} + \underline{b'}^2 \underline{d} + c^3 \underline{a'}$ $\mathbb{P} dc_0 + c_1$, i.e. $\mathbb{P} c_1 d_0$
i.e. My servants never say "shpoonj."

5.

$ad_0 + cd'_0 + b_1 a'_0$; $\underline{a}^1 \underline{d} + c^2 \underline{d'} + b^3 \underline{a'}$ $\mathbb{P} cb_0 + b_1$, i.e. $\mathbb{P} b_1 c_0$
i.e. My poultry are not officers.

6.

$c_1 a'_0 + c'b_0 + da_0$; $\underline{c}^1 \underline{a'} + \underline{c'}^2 \underline{b} + d^3 \underline{a}$ $\mathbb{P} bd_0$
i.e. None of *your* sons are fit to serve on a jury.

7.

$cb_0 + da_0 + b'_1 a'_0$; $c^1 \underline{b} + \underline{b'}^3 \underline{a'} + d^2 \underline{a}$ $\mathbb{P} cd_0$
i.e. No pencils of mine are sugarplums.

8.

$cb'_0 + d_1 a'_0 + ba_0$; $c^1 \underline{b'} + \underline{b}^3 \underline{a} + d^2 \underline{a'}$ $\mathbb{P} cd_0 + d_1$, i.e. $\mathbb{P} d_1 c_0$
i.e. Jenkins is inexperienced.

9.

$cd_0 + d'a_0 + c'b_0$; $\underline{c}^1 \underline{d} + \underline{d'}^2 \underline{a} + \underline{c'}^3 \underline{b}$ $\mathbb{P} ab_0$
i.e. No comet has a curly tail.

10.

$d'_1 c_0 + ba_0 + a'_1 d_0$; $\underline{d'}^1 \underline{c} + \underline{a'}^3 \underline{d} + b^2 \underline{a}$ $\mathbb{P} cb_0$
i.e. No hedgehog takes in the *Times*.

11.

$b_1 a'_0 + c_1 b'_0 + ad_0$; $\underline{b}^1 \underline{a'} + c^2 \underline{b'} + \underline{a}^3 \underline{d}$ $\mathbb{P} cd_0 + c_1$, i.e. $\mathbb{P} c_1 d_0$
i.e. This dish is unwholesome.

12.

$c_1 d'_0 + ba'_0 + d_1 a_0$; $c^1 \underline{d'} + \underline{d}^3 \underline{a} + b^2 \underline{a'}$ $\mathbb{P} cb_0 + c_1$, i.e. $\mathbb{P} c_1 b_0$
i.e. Showy talkers are not really well-informed.

13.

$a_1 d'_0 + bc_0 + c'_1 d_0$; $a^1 \underline{d'} + \underline{c'}^3 \underline{d} + b^2 \underline{c}$ $\mathbb{P} ab_0 + a_1$, i.e. $\mathbb{P} a_1 b_0$
i.e. All humming-birds are small.

Ex. 112, 113; Ans. 131.]

14.

$$\overset{1}{c}b_0 + \overset{2}{a_1}d'_0 + \overset{3}{c}a'_0; \quad \overset{1}{c}b + \overset{3}{c}a' + \overset{2}{a}d' \mathbb{P} ba'_0$$

i.e. No one with a hooked nose ever fails to make money.

15.

$$\overset{1}{a}b_0 + \overset{2}{c}d_0 + \overset{3}{a'}a_0; \quad \overset{1}{a}b + \overset{3}{a'}a + \overset{2}{c}d \mathbb{P} bc_0$$

i.e. No acrobatic feat, which involves turning a quadruple somersault, is ever attempted in a circus.

16.

$$\overset{1}{b_1}c'_0 + \overset{2}{d'}a_0 + \overset{3}{a'}c_0; \quad \overset{1}{b}c' + \overset{3}{a'}c + \overset{2}{a'}a \mathbb{P} ba'_0 + b_1, \text{ i.e. } \mathbb{P} b_1a'_0$$

i.e. My gardener is very old.

17.

$$\overset{1}{b_1}a'_0 + \overset{2}{b'_1}d_0 + \overset{3}{c}a_0; \quad \overset{1}{b}a' + \overset{2}{b'}d + \overset{3}{c}a \mathbb{P} dc_0$$

i.e. No gray ducks in this village wear lace collars.

18.

$$\overset{1}{a_1}b'_0 + \overset{2}{d}c_0 + \overset{3}{a'_1}d'_0; \quad \overset{1}{a}b' + \overset{3}{a'}d' + \overset{2}{d}c \mathbb{P} b'_c_0$$

i.e. No M.P. should ride in a donkey-race, unless he has perfect self-command.

19.

$$\overset{1}{c_1}a'_0 + \overset{2}{b}c'_0 + \overset{3}{a}d_0; \quad \overset{1}{c}a' + \overset{2}{b}c' + \overset{3}{a}d \mathbb{P} bd_0$$

i.e. No jug in this cupboard will hold water.

20.

$$\overset{1}{d}c'_0 + \overset{2}{a_1}b'_0 + \overset{3}{b}c_0; \quad \overset{1}{d}c' + \overset{3}{b}c + \overset{2}{a}b' \mathbb{P} da_0 + a_1, \text{ i.e. } \mathbb{P} a_1d_0$$

i.e. Guinea-pigs never really appreciate Beethoven.

21.

$$\overset{1}{b'_1}b'_0 + \overset{2}{c_1}b_0 + \overset{3}{c'}a_0; \quad \overset{1}{d'}b' + \overset{2}{c}b + \overset{3}{c'}a \mathbb{P} d'a_0 + d'_1, \text{ i.e. } \mathbb{P} d'_1a_0$$

i.e. Puppies, that will not lie still, never care to do worsted-work.

22.

$$\overset{1}{b'_1}d_0 + \overset{2}{c_1}d'_0 + \overset{3}{a}b_0; \quad \overset{1}{b'}d + \overset{2}{c}d' + \overset{3}{a}b \mathbb{P} ca_0 + c_1, \text{ i.e. } \mathbb{P} c_1a_0$$

i.e. These apples were grown in the sun.

23.

$$\overset{1}{a}d_0 + \overset{2}{b'}c_0 + \overset{3}{a'}b_0; \quad \overset{1}{a}d + \overset{3}{a'}b + \overset{2}{b'}c \mathbb{P} dc_0$$

i.e. No goods in this shop, that are still on sale, may be carried away.

(Ex. 113-115 ; Ans. 131, 132.)

24.

$$a_1 d'_0 + b'_1 c_0 + ba'_0; \quad \underline{a} d' + \underline{b} \underline{a}' + \underline{b}' c \text{ } \mathbb{P} d' c_0$$

.e. No scentless flowers please me.

25.

$$bd'_0 + a_1 c'_0 + a' d_0; \quad b \underline{d}' + \underline{a}' \underline{d} + \underline{a} c' + \mathbb{P} bc'_0$$

.e. No name in this list is unmelodious.

26.

$$d_1 e_0 + c' a_0 + b_1 e'_0 + c_1 d'_0; \quad \underline{d} \underline{e} + \underline{b} \underline{e}' + \underline{c} \underline{d}' + \underline{c}' a \text{ } \mathbb{P} ba_0 + b_1, \text{ i.e. } \mathbb{P} b_1 a_0$$

i.e. An egg of the Great Awk is not to be had for a song.

27.

$$c'b_0 + a_1 d'_0 + d_1 e'_0 + a' c_0; \quad \underline{c}' b + \underline{a}' \underline{c} + \underline{a} d' + \underline{d} e' \text{ } \mathbb{P} be'_0$$

i.e. No books sold here have gilt edges unless they are priced at 5s. and upwards.

28.

$$e'_1 b'_0 + c_1 a_0 + e_1 d'_0 + b_1 a'_0; \quad \underline{e}' \underline{b}' + \underline{e} d' + \underline{b} \underline{a}' + \underline{c} \underline{a} \text{ } \mathbb{P} d' c_0 + c_1, \text{ i.e. } \mathbb{P} c_1 d'_0$$

i.e. When you cut your finger, you will find Tincture of Calendula useful.

29.

$$d_1 a_0 + e' c_0 + b_1 a'_0 + d' e_0; \quad \underline{d} \underline{a} + \underline{b} \underline{a}' + \underline{d}' \underline{e} + \underline{e}' c \text{ } \mathbb{P} bc_0 + b_1, \text{ i.e. } \mathbb{P} b_1 c_0$$

i.e. All the gluttons in my family are unhealthy.

30.

$$c'_1 b'_0 + a_1 e'_0 + d_1 b'_0 + a'_1 c_0; \quad \underline{c}' \underline{b}' + \underline{d} \underline{b}' + \underline{a}' \underline{c} + \underline{a} e' \text{ } \mathbb{P} de'_0 + d_1, \text{ i.e. } \mathbb{P} d_1 e'_0$$

i.e. All the romances in this library are well-written.

31.

$$e' d_0 + c' a_0 + eb_0 + d' c_0; \quad \underline{e}' \underline{d} + \underline{e} b + \underline{d}' \underline{c} + \underline{c}' a \text{ } \mathbb{P} ba_0$$

i.e. No bird in this aviary lives on mince-pies.

32.

$$d'_1 c'_0 + e_1 a'_0 + c_1 b_0 + e' d_0; \quad \underline{d}' \underline{c}' + \underline{e} b + \underline{e}' \underline{d} + \underline{e} a' \text{ } \mathbb{P} ba'_0$$

i.e. No plum-pudding, that has not been boiled in a cloth, can be distinguished from soup.

33

$$ea_0 + b_1 d'_0 + a'_1 c_0 + e' b'_0; \quad \underline{e} \underline{a} + \underline{a}' \underline{c} + \underline{e}' \underline{b}' + \underline{b} \underline{a}' \text{ } \mathbb{P} cd'_0$$

i.e. None but red-haired boys learn Greek in this school

Ex. 115, 116; Ans. 132.

34.

$\overset{1}{b_1} \overset{2}{d_0} \dagger \overset{3}{a} \overset{4}{c'_0} \dagger \overset{1}{c_1} \overset{3}{d'_0} \dagger \overset{4}{c_1} \overset{2}{b'_0}$; $\underline{b} \underline{d} \dagger \underline{e} \underline{d'} \dagger \underline{c} \underline{b'} \dagger \underline{a} \underline{c'} \vdash \mathbb{P} \text{ea}_0 \dagger c_1$, i.e. $\mathbb{P} c_1 a_0$
i.e. Wedding-cake always disagrees with me.

35.

$\overset{1}{db_0} \dagger \overset{2}{a_1} \overset{3}{e'_0} \dagger \overset{4}{ec_0} \overset{4}{d'_1} \overset{2}{a'_0}$; $\underline{d} \underline{b} \dagger \underline{d'} \underline{a'} \dagger \underline{a} \underline{e'} \dagger \underline{e} \underline{c} \vdash \mathbb{P} bc_0$
i.e. *I* have never come across a mermaid at sea.

36.

$\overset{1}{ad_0} \dagger \overset{2}{e'_1} \overset{3}{b'_0} \dagger \overset{4}{c_1} \overset{4}{d'_0} \dagger \overset{2}{c_1} \overset{2}{a'_0}$; $\underline{a} \underline{d} \dagger \underline{c} \underline{d'} \dagger \underline{e} \underline{a'} \dagger \underline{e'} \underline{b'} \vdash \mathbb{P} cb'_0 \dagger c_1$, i.e. $\mathbb{P} c_1 b'_0$
i.e. Any discussion, that goes on while Tomkins is in the chair, endangers the peacefulness of our Debating-Club.

37.

$\overset{1}{c_1} \overset{2}{a'_0} \dagger \overset{3}{h'_0} \overset{4}{b_0} \dagger \overset{5}{ae_0} \dagger \overset{4}{d_1} \overset{2}{c'_0} \dagger \overset{5}{h_1} \overset{2}{e'_0}$; $\underline{c} \underline{a'} \dagger \underline{a} \underline{e} \dagger \underline{d} \underline{c'} \dagger \underline{h} \underline{e'} \dagger \underline{h'} \underline{b} \vdash \mathbb{P} db_0 \dagger d_1$,
i.e. $\mathbb{P} d_1 b_0$
i.e. All *my* friends dine at the lower table.

38.

$\overset{1}{d'_1} \overset{2}{b'_0} \dagger \overset{3}{ha'_0} \dagger \overset{4}{c_1} \overset{5}{e_0} \dagger \overset{4}{d_1} \overset{2}{e'_0} \dagger \overset{5}{ba_0}$; $\underline{d'} \underline{b'} \dagger \underline{d} \underline{e'} \dagger \underline{c} \underline{e} \dagger \underline{b} \underline{a} \dagger \underline{h} \underline{a'} \vdash \mathbb{P} ch_0 \dagger c_1$,
i.e. $\mathbb{P} c_1 h_0$
i.e. Rainbows are not worth writing odes to.

39.

$\overset{1}{c'_1} \overset{2}{d_0} \dagger \overset{3}{h_1} \overset{4}{e'_0} \dagger \overset{5}{c_1} \overset{4}{a'_0} \dagger \overset{5}{h'_0} \overset{2}{b_0} \dagger \overset{5}{c_1} \overset{2}{d'_0}$; $\underline{c'} \underline{d} \dagger \underline{c} \underline{a'} \dagger \underline{e} \underline{d'} \dagger \underline{h} \underline{e'} \dagger \underline{h'} \underline{b} \vdash \mathbb{P} a'b_0$
i.e. No pawnbroker is dishonest.

40.

$\overset{1}{ad'_0} \dagger \overset{2}{c'h_0} \dagger \overset{3}{c_1} \overset{4}{a'_0} \dagger \overset{5}{db_0} \dagger \overset{5}{e'_0} \overset{2}{c_0}$; $\underline{a} \underline{d'} \dagger \underline{e} \underline{a'} \dagger \underline{d} \underline{b} \dagger \underline{e'} \underline{c} \dagger \underline{c'} \underline{h} \vdash \mathbb{P} bh_0$
i.e. No kitten with green eyes will play with a gorilla.

41.

$\overset{1}{ce'_0} \dagger \overset{2}{b'a'_0} \dagger \overset{3}{h_1} \overset{4}{d'_0} \dagger \overset{5}{ae_0} \dagger \overset{5}{bd_0}$; $\underline{c} \underline{e'} \dagger \underline{a} \underline{e} \dagger \underline{b'} \underline{a'} \dagger \underline{b} \underline{d} \dagger \underline{h} \underline{d'} \vdash \mathbb{P} ch_0 \dagger h_1$,
i.e. $\mathbb{P} h_1 c_0$
i.e. All *your* poems are uninteresting.

42.

$\overset{1}{b'_1} \overset{2}{a'_0} \dagger \overset{3}{db_0} \dagger \overset{4}{h_1} \overset{5}{e'_0} \dagger \overset{5}{e_1} \overset{2}{c_0} \dagger \overset{4}{a_1} \overset{2}{h'_0}$; $\underline{b'} \underline{a'} \dagger \underline{d} \underline{b} \dagger \underline{a} \underline{h'} \dagger \underline{h} \underline{e'} \dagger \underline{e} \underline{c} \vdash \mathbb{P} dc_0$
i.e. None of my peaches have been grown in a hothouse.

43.

$\overset{1}{c'_1} \overset{2}{e_0} \dagger \overset{3}{ah_0} \dagger \overset{4}{db_0} \dagger \overset{5}{e'_1} \overset{4}{a'_0} \dagger \overset{5}{cb'_0}$; $\underline{c'} \underline{e} \dagger \underline{e'} \underline{a'} \dagger \underline{a} \underline{h} \dagger \underline{c} \underline{b'} \dagger \underline{d} \underline{b} \vdash \mathbb{P} hd_0$
i.e. No Mandarin ever reads Hogg's poems.

44.

$${}^1be_0 + {}^2h_1d'_0 + {}^3b'_1c'_0 + {}^4a'e'_0 + {}^5d_1c_0; \quad {}^1\underline{b}\underline{e} + {}^3\underline{b}'\underline{c}' + {}^4\underline{a}'\underline{e}' + {}^5\underline{d}\underline{c} + {}^2h\underline{d}' \text{ } \mathbb{P} \text{ } a'h_0 + h_1, \\ \text{i.e. } \mathbb{P} \text{ } h_1a'_0$$

i.e. My writing-desk is full of live scorpions.

45.

$${}^1c_1b'_0 + {}^2a'd_0 + {}^3c_1h'_0 + {}^4e'a_0 + {}^5d'h_0; \quad {}^1\underline{e}\underline{b}' + {}^4\underline{e}'\underline{a} + {}^2\underline{a}'\underline{d} + {}^5\underline{d}'\underline{h} + {}^3c\underline{h}' \text{ } \mathbb{P} \text{ } b'_c_0 + c_1, \\ \text{i.e. } \mathbb{P} \text{ } c_1b'_0$$

i.e. Shakespeare was clever.

46.

$${}^1c'_1h'_0 + {}^2c_1a_0 + {}^3bd_0 + {}^4a'_1h_0 + {}^5d'_c_0; \quad {}^1\underline{c}'\underline{h}' + {}^4\underline{a}'\underline{h} + {}^2\underline{e}\underline{a} + {}^5\underline{d}'\underline{c} + {}^3b\underline{d} \text{ } \mathbb{P} \text{ } cb_0 + c_1, \\ \text{i.e. } \mathbb{P} \text{ } c_1b_0$$

i.e. These Sorites-examples are difficult.

47.

$${}^1k'_1e_0 + {}^2c_1h_0 + {}^3a_1b'_0 + {}^4kd_0 + {}^5h'b_0 + {}^6a'_1c'_0; \quad {}^1\underline{k}'\underline{e} + {}^4\underline{k}\underline{d} + {}^6\underline{a}'\underline{e} + {}^3\underline{a}\underline{b}' + {}^5\underline{h}'\underline{b} + {}^2c\underline{h} \\ \text{ } \mathbb{P} \text{ } dc_0 + c_1, \text{ i.e. } \mathbb{P} \text{ } c_1d_0$$

i.e. Donkeys are not easy to swallow.

48.

$${}^1ab'_0 + {}^2h'_c_0 + {}^3d_1e_0 + {}^4b_1c'_0 + {}^5a'_k_0 + {}^6c'_1h_0; \quad {}^1\underline{a}\underline{b}' + {}^4\underline{b}\underline{c}' + {}^2\underline{h}'\underline{c} + {}^5\underline{a}'\underline{k} + {}^6\underline{e}'\underline{h} + {}^3d\underline{e} \\ \text{ } \mathbb{P} \text{ } kd_0 + d_1, \text{ i.e. } \mathbb{P} \text{ } d_1k_0$$

i.e. Opium-eaters never wear white kid gloves.

49.

$${}^1a'h_0 + {}^2c'_k_0 + {}^3a_1d'_0 + {}^4c_1h'_0 + {}^5b_1k'_0 + {}^6c_1e'_0; \quad {}^1\underline{a}'\underline{h} + {}^3\underline{a}\underline{d}' + {}^4\underline{e}\underline{h}' + {}^6\underline{c}\underline{e}' + {}^2\underline{c}'\underline{k} + {}^5b\underline{k}' \\ \text{ } \mathbb{P} \text{ } d'b_0 + b_1, \text{ i.e. } \mathbb{P} \text{ } b_1d'_0$$

i.e. All the English pictures here are painted in oils.

50.

$${}^1c_1h'_0 + {}^2c_1b'_0 + {}^3\underline{k}'\underline{a}_0 + {}^4d_1c'_0 + {}^5h_1a'_0 + {}^6k_1c'_0; \quad {}^1\underline{e}\underline{h}' + {}^4\underline{d}\underline{e}' + {}^5\underline{h}\underline{a}' + {}^3\underline{k}'\underline{a} + {}^6\underline{k}\underline{c}' + {}^2\underline{c}\underline{b}' \\ \text{ } \mathbb{P} \text{ } db'_0 + d_1, \text{ i.e. } \mathbb{P} \text{ } d_1b'_0$$

i.e. All the animals in the yard gnaw bones.

51.

$${}^1a'_1h'_0 + {}^2ck_0 + {}^3b_1d'_0 + {}^4k'h_0 + {}^5ca + {}^6d_1c'_0; \quad {}^1\underline{a}'\underline{h}' + {}^4\underline{k}'\underline{h} + {}^2\underline{c}\underline{k} + {}^5\underline{e}\underline{a} + {}^6\underline{d}\underline{c}' + {}^3b\underline{d}' \\ \text{ } \mathbb{P} \text{ } cb_0 + b_1, \text{ i.e. } \mathbb{P} \text{ } b_1c^0$$

i.e. Bathing-machines are never made of mother-of-pearl.

52.

$${}^1ac_0 + {}^2k_1b'_0 + {}^3ch_0 + {}^4d_1a'_0 + {}^5h'_c_0 + {}^6k'_1e'_0; \quad {}^1\underline{a}\underline{c} + {}^4\underline{d}\underline{a}' + {}^5\underline{h}'\underline{c}' + {}^3\underline{e}\underline{h} + {}^6\underline{k}'\underline{e}' + {}^2\underline{k}\underline{b}' \\ \text{ } \mathbb{P} \text{ } db'_0 + d_1, \text{ i.e. } \mathbb{P} \text{ } d_1b'_0$$

i.e. A good husband always comes home for his tea.

APPENDIX,

ADDRESSED TO TEACHERS.

§ 1.

Introductory.

THERE are several matters, too hard to discuss with *Learners*, which nevertheless need to be explained to any *Teachers*, into whose hands this book may fall, in order that they may thoroughly understand what my Symbolic Method *is*, and in what respects it differs from the many other Methods already published.

These matters are as follows :—

The “Existential Import” of Propositions.

The use of “is-not” (or “are-not”) as a Copula.

The theory “two Negative Premisses prove nothing.”

Euler’s Method of Diagrams.

Venn’s Method of Diagrams.

My Method of Diagrams.

The Solution of a Syllogism by various Methods.

My Method of treating Syllogisms and Sorites.

Some account of Parts II, III.

§ 2.

The “Existential Import” of Propositions.

The writers, and editors, of the Logical text-books which run in the ordinary grooves—to whom I shall hereafter refer by the (I hope inoffensive) title “The Logicians”—take, on this subject, what seems to me to be a more humble position than is at all necessary. They speak of the Copula of a Proposition “with bated breath”, almost as if it were a living, conscious Entity, capable of declaring for itself what it chose to mean and that we, poor human creatures, had nothing to do but to ascertain *what* was its sovereign will and pleasure, and submit to it.

In opposition to this view, I maintain that any writer of a book is fully authorised in attaching any meaning he likes to any word or phrase he intends to use. If I find an author saying, at the beginning of his book, "Let it be understood that by the word '*black*' I shall always mean '*white*', and that by the word '*white*' I shall always mean '*black*'," I meekly accept his ruling, however injudicious I may think it.

And so, with regard to the question whether a Proposition is or is not to be understood as asserting the existence of its Subject, I maintain that every writer may adopt his own rule, provided of course that it is consistent with itself and with the accepted facts of Logic.

Let us consider, one by one, the various views that may *logically* be held, and thus settle which of them may *conveniently* be held; after which I shall hold myself free to declare which of them *I* intend to hold.

The *kinds* of Proposition, to be considered, are those that begin with "some", with "no", and with "all". These are usually called Propositions "in *I*", "in *E*", and "in *A*".

First, then, a Proposition in *I* may be understood as asserting, or else as *not* asserting, the existence of its Subject. (By "existence" I mean of course whatever kind of existence suits its nature. The two Propositions, "*dreams* exist" and "*drums* exist", denote two totally different kinds of "existence". A *dream* is an aggregate of ideas, and exists only in the *mind of a dreamer*: whereas a *drum* is an aggregate of wood and parchment, and exists in the *hands of a drummer*.) Let us first consider the case of its "asserting" (by which I mean "asserting the existence of its Subject").

Here, of course, we must regard a Proposition in *A* as making the *same* assertion, since it necessarily *contains* a Proposition in *I*.

We now have *I* and *A* "asserting". Does this leave us free to make what supposition we choose as to *E*? My answer is "No. We are tied down to the supposition that *E* does *not* assert." This can be proved as follows:—

If possible, let *E* "assert". Then (taking *x*, *y*, and *z* to represent Attributes) we see that, if the Proposition "No *xy* are *z*" be true, some things exist with the Attributes *x* and *y*: i.e. "Some *x* are *y*."

Also we know that, if the Proposition "Some xy are z " be true, the same result follows.

But these two Propositions are Contradictories, so that one or other of them *must* be true. Hence this result is *always* true: i.e. the Proposition "Some x are y " is *always* true!

Quod est absurdum.

We see, then, that the supposition " I asserts" necessarily leads to " A asserts, but E does not". And this is the *first* of the various views that may conceivably be held.

Next, let us suppose that I does *not* "assert." And, along with this, let us take the supposition that E *does* "assert."

Hence the Proposition "No x are y " means "Some x exist, and none of them are y ": i.e. "*all* of them are *not-y*," which is a Proposition in A . We also know, of course, that the Proposition "All x are *not-y*" proves "No x are y ." Now two Propositions, each of which proves the other, are *equivalent*. Hence every Proposition in A is equivalent to one in E , and therefore "*asserts*".

Hence our *second* conceivable view is " E and A assert, but I does not."

This view does not seem to involve any necessary contradiction with itself or with the accepted facts of Logic. But, when we come to *test* it, as applied to the actual *facts* of life, we shall find I think, that it fits in with them so badly that its adoption would be, to say the least of it, singularly inconvenient for ordinary folk.

Let me record a little dialogue I have just held with my friend Jones, who is trying to form a new Club, to be regulated on strictly *Logical* principles.

Author. "Well, Jones! Have you got your new Club started yet?"

Jones (rubbing his hands). "You'll be glad to hear that some of the Members (mind, I only say '*some*') are millionaires! Rolling in gold, my boy!"

Author. "That sounds well. And how many Members have entered?"

Jones (staring). "None at all. We haven't got it started yet. What makes you think we have?"

Author. "Why, I thought you said that some of the Members——"

Jones (contemptuously). "You don't seem to be aware that we're working on strictly *Logical* principles. A *Particular Proposition* does not assert the existence of its Subject. I merely meant to say that we've made a Rule not to admit any Members till we have at least *three* Candidates whose incomes are over ten thousand a year."

Author. "Oh, *that's* what you meant, is it! Let's hear some more of your Rules."

Jones. "Another is, that no one, who has been convicted seven times of forgery, is admissible."

Author. "And here, again, I suppose you don't mean to assert there are any such convicts in existence!"

Jones. "Why, that's exactly what I *do* mean to assert! Don't you know that a Universal Negative *asserts* the existence of its Subject! *Of course* we didn't make that Rule till we had satisfied ourselves that there are several such convicts now living."

The Reader can now decide for himself how far this *second* conceivable view would fit in with the facts of life. He will, I think, agree with me that Jones' view, of the 'Existential Import' of Propositions, would lead to some inconvenience.

Thirdly, let us suppose that neither *I* nor *E* "asserts".

Now the supposition that the two Propositions, "Some *x* are *y*" and "No *x* are not-*y*", do not "assert", necessarily involves the supposition that "All *x* are *y*" does not "assert", since it would be absurd to suppose that they assert, when combined, more than they do when taken separately.

Hence the *third* (and last) of the conceivable views is that neither *I*, nor *E*, nor *A*, "asserts".

The advocates of this third view would interpret the Proposition "Some *x* are *y*" to mean "If there *were* any *x* in existence, some of them *would* be *y*"; and so with *E* and *A*.

It admits of proof that this view, as regards *A*, conflicts with the accepted facts of Logic.

Let us take the Syllogism *Darapti*, which is universally accepted as valid. Its form is

"All *m* are *x* ;

All *m* are *y*.

∴ Some *y* are *x*."

This they would interpret as follows :—

“If there were any m in existence, all of them would be x ;
 If there were any m in existence, all of them would be y .
 \therefore If there were any y in existence, some of them
 would be x ”.

That this Conclusion does *not* follow has been so briefly and clearly explained by Mr. Keynes (in his “Formal Logic”, dated 1894, pp. 356, 357), that I prefer to quote his words :—

“*Let no proposition imply the existence either of its subject or of its predicate.*

“Take, as an example, a syllogism in *Darapti* :—

‘ All M is P ,
 All M is S ,
 \therefore Some S is P .’

“Taking S , M , P , as the minor, middle, and major terms respectively, the conclusion will imply that, if there is any S , there is some P . Will the premisses also imply this? If so, then the syllogism is valid ; but not otherwise.

“The conclusion implies that if S exists P exists ; but, consistently with the premisses, S may be existent while M and P are both non-existent. An implication is, therefore, contained in the conclusion, which is not justified by the premisses”.

This seems to *me* entirely clear and convincing. Still, “to make sicker”, I may as well throw the above (*soi-disant*) Syllogism into a concrete form, which will be within the grasp of even a *non*-logical Reader.

Let us suppose that a Boys’ School has been set up, with the following system of Rules :—

“All boys in the First (the highest) Class are to do French, Greek, and Latin. All in the Second Class are to do Greek only. All in the Third Class are to do Latin only.”

Suppose also that there *are* boys in the Third Class, and in the Second ; but that no boy has yet risen into the First.

It is evident that there are no boys in the School doing French : still we know, by the Rules, what would happen if *there were any*.

We are authorised, then, by the *Data*, to assert the following two Propositions :—

“ If there were any boys doing French, all of them would be doing Greek ;

If there were any boys doing French, all of them would be doing Latin ”.

And the Conclusion, according to “The Logicians ” would be -

“ If there were any boys doing Latin, some of them would be doing Greek ”.

Here, then, we have two *true* Premisses and a *false* Conclusion (since we know that there *are* boys doing Latin, and that *none* of them are doing Greek). Hence the argument is *invalid*.

Similarly it may be shown that this “non-existential” interpretation destroys the validity of *Disamis*, *Datissi*, and *Felapton*.

It is always sad to see a batsman knock down his own wicket : one pities him, as a man and a brother, but, as a *cricketer*, one can but pronounce him “ Out ! ”

We see, then, that, among all the conceivable views, there are only *two* which can *logically* be held, viz.

I and *A* “ assert ”, but *E* does not.

E and *A* “ assert ”, but *I* does not.

The *second* of these I have shown to involve great practical inconvenience.

The *first* is the one adopted in this book. (See p. 20.)

§ 3

The use of “is-not” (or “are-not”) as a Copula.

Is it better to say “John *is-not* in-the-house” or “John *is not-in-the-house*”? “Some of my acquaintances *are-not* men-I-should-like-to-be-seen-with” or “Some of my acquaintances *are* men-I-should-not-like-to-be-seen-with”? That is the sort of question we have now to discuss.

This is no question of Logical Right and Wrong: it is merely a matter of *taste*, since the two forms mean exactly the same thing. And here, again, "The Logicians" seem to me to take much too humble a position. When they are putting the final touches to the grouping of their Proposition, just before the curtain goes up, and when the Copula——always a rather fussy 'heavy father', asks them "Am *I* to have the 'not', or will you tack it on to the Predicate?" they are much too ready to answer, like the subtle cab-driver, "Leave it to *you*, Sir!" The result seems to be, that the grasping Copula constantly gets a "not" that had better have been merged in the Predicate, and that Propositions are differentiated which had better have been recognised as precisely similar. Surely it is simpler to treat "Some men are Jews" and "Some men are Gentiles" as being, both of them, *affirmative* Propositions, instead of translating the latter into "Some men are-not Jews", and regarding it as a *negative* Proposition?

The fact is, "The Logicians" have somehow acquired a perfectly *morbid* dread of negative Attributes, which makes them shut their eyes, like frightened children, when they come across such terrible Propositions as "All not-*x* are *y*"; and thus they exclude from their system many very useful forms of Syllogisms.

Under the influence of this unreasoning terror, they plead that, in Dichotomy by Contradiction, the *negative* part is too large to deal with, so that it is better to regard each Thing as either included in, or excluded from, the *positive* part. I see no force in this plea: and the facts often go the other way. As a personal question, dear Reader, if *you* were to group your acquaintances into the two Classes, men that you *would* like to be seen with, and men that you would *not* like to be seen with, do you think the latter group would be so *very* much the larger of the two?

For the purposes of Symbolic Logic, it is so *much* the most convenient plan to regard the two sub-divisions, produced by Dichotomy, on the *same* footing, and to say, of any Thing, either that it "is" in the one, or that it "is" in the other, that I do not think any Reader of this book is likely to demur to my adopting that course.

§ 4.

The theory that "two Negative Premisses prove nothing".

This I consider to be *another* craze of "The Logicians", fully as morbid as their dread of a negative Attribute.

It is, perhaps, best refuted by the method of *Instantia Contraria*.

Take the following Pairs of Premisses :—

"None of my boys are conceited ;
None of my girls are greedy".

"None of my boys are clever ;
None but a clever boy could solve this problem".

"None of my boys are learned ;
Some of my boys are not choristers".

(This last Proposition is, in *my* system, an *affirmative* one, since *I* should read it "are not-choristers" ; but, in dealing with "The Logicians," I may fairly treat it as a *negative* one, since *they* would read it "are-not choristers".)

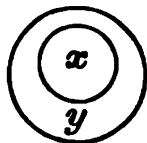
If you, dear Reader, declare, after full consideration of these Pairs of Premisses, that you cannot deduce a Conclusion from *any* of them——why, all I can say is that, like the Duke in *Patience*, you "will have to be contented with our heart-felt sympathy" !

§ 5.

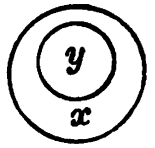
Euler's Method of Diagrams.

Diagrams seem to have been used, at first, to represent *Propositions* only. In Euler's well-known Circles, each was supposed to contain a Class, and the Diagram consisted of two Circles, which exhibited the relations, as to inclusion and exclusion, existing between the two Classes.

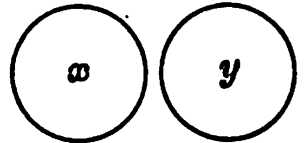
Thus, the Diagram, here given, exhibits the two Classes, whose respective Attributes are x and y , as so related to each other that the following Propositions are all simultaneously true :—"All x are y ", "No x are not- y ", "Some x are y ", "Some y are not- x ", "Some not- y are not- x ", and, of course, the Converses of the last four.



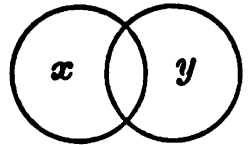
Similarly, with this Diagram, the following Propositions are true:—"All y are x ", "No y are not- x ", "Some y are x ", "Some x are not- y ", "Some not- x are not- y ", and, of course, the Converses of the last four.



Similarly, with this Diagram, the following are true:—"All x are not- y ", "All y are not- x ", "No x are y ", "Some x are not- y ", "Some y are not- x ", "Some not- x are not- y ", and the Converses of the last four.



Similarly, with this Diagram, the following are true:—"Some x are y ", "Some x are not- y ", "Some not- x are y ", "Some not- x are not- y ", and, of course, their four Converses.



It is amusing to note that *all* Euler's Diagrams assert the Proposition "Some not- x are not- y ." Apparently it never occurred to Euler that it might *sometimes* fail to be true!

Now, to represent "All x are y ", the *first* of these Diagrams would suffice. Similarly, to represent "No x are y ", the *third* would suffice. But to represent any *Particular* Proposition, at least *three* Diagrams would be needed (in order to include all the possible cases), and, for "Some not- x are not- y ", all the *four*.

Let us represent "not- x " by " x' ".

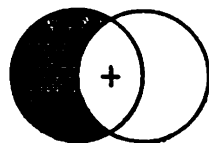
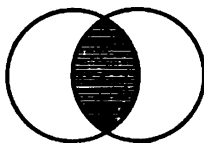
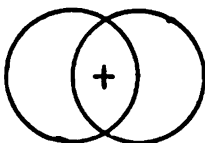
§ 6.

Venn's Method of Diagrams.

Mr. Venn's Method of Diagrams is a great advance on the above Method.

He uses the last of the above Diagrams to represent *any* desired relation between x and y , by simply shading a Compartment known to be *empty*, and placing a $+$ in one known to be *occupied*.

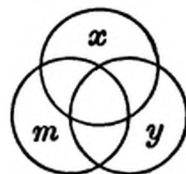
Thus, he would represent the three Propositions "Some x are y ", "No x are y ", and "All x are y ", as follows:—



It will be seen that, of the *four* Classes, whose peculiar Sets of Attributes are xy , xy' , $x'y$, and $x'y'$, only *three* are here provided with closed Compartments, while the *fourth* is allowed the rest of the Infinite Plane to range about in !

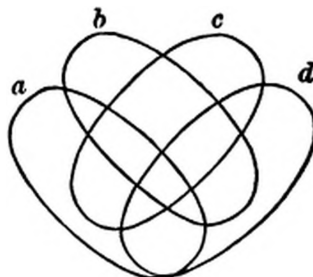
This arrangement would involve us in very serious trouble, if we ever attempted to represent "No x' are y' ." Mr. Venn *once* (at p. 281) encounters this awful task ; but evades it, in a quite masterly fashion, by the simple foot-note "We have not troubled to shade the outside of this diagram" !

To represent *two* Propositions (containing a common Term) *together*, a *three-letter* Diagram is needed. This is the one used by Mr. Venn.

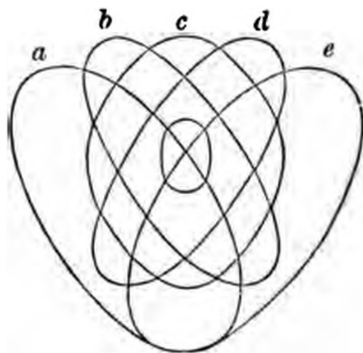


Here, again, we have only *seven* closed Compartments, to accommodate the *eight* Classes whose peculiar Sets of Attributes are xym , xym' , &c.

"With four terms in request," Mr. Venn says, "the most simple and symmetrical diagram seems to me that produced by making four ellipses intersect one another in the desired manner". This, however, provides only *fifteen* closed compartments.



For *five* letters, "the simplest diagram I can suggest," Mr. Venn says, "is one like this (the small ellipse in the centre is to be regarded as a portion of the *outside* of c ; i.e. its four component portions are inside b and d but are no part of c). It must be admitted that such a diagram is not quite so simple to draw as one might wish it to be ; but then consider what the alternative is if one undertakes to deal with five terms and all their combinations—*nothing* short of the disagreeable task of writing out, or in some *way putting* before us, all the 32 combinations involved."



This Diagram gives us 31 closed compartments.

For *six* letters, Mr. Venn suggests that we might use *two* Diagrams, like the above, one for the *f*-part, and the other for the not-*f*-part, of all the other combinations. "This", he says, "would give the desired 64 subdivisions." This, however, would only give 62 closed Compartments, and *one* infinite area, which the two Classes, $a'b'c'd'e'f$ and $a'b'c'd'e'f'$, would have to share between them.

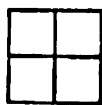
Beyond *six* letters Mr. Venn does not go.

§ 7.

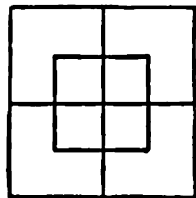
My Method of Diagrams.

My Method of Diagrams *resembles* Mr. Venn's, in having separate Compartments assigned to the various Classes, and in marking these Compartments as *occupied* or as *empty*; but it *differs* from his Method, in assigning a *closed* area to the *Universe of Discourse*, so that the Class which, under Mr. Venn's liberal sway, has been ranging at will through Infinite Space, is suddenly dismayed to find itself "cabin'd, cribb'd, confined", in a limited Cell like any other Class! Also I use *rectilinear*, instead of *curvilinear*, Figures; and I mark an *occupied* Cell with a 'I' (meaning that there is at least *one* Thing in it), and an *empty* Cell with a 'O' (meaning that there is *no* Thing in it).

For *two* letters, I use this Diagram, in which the North Half is assigned to ' x ', the South to ' $\text{not-}x$ ' (or ' x' '), the West to y , and the East to y' . Thus the N.W. Cell contains the xy -Class, the N.E. Cell the xy' -Class, and so on.

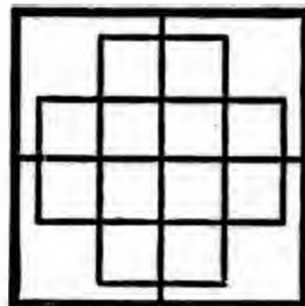


For *three* letters, I subdivide these four Cells, by drawing an *Inner Square*, which I assign to m , the *Outer Border* being assigned to m' . I thus get the *eight* Cells that are needed to accommodate the eight Classes, whose peculiar Sets of Attributes are xym , xym' , &c.

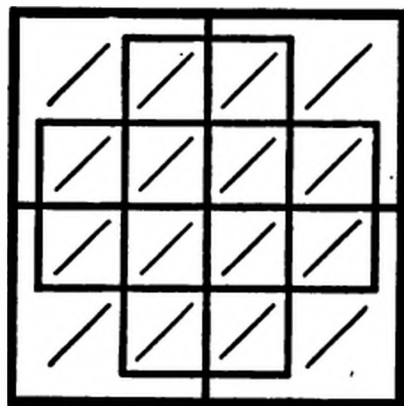


This last Diagram is the most complex that I use in the *Elementary* Part of my 'Symbolic Logic.' But I may as well take this opportunity of describing the more complex ones which will appear in Part II.

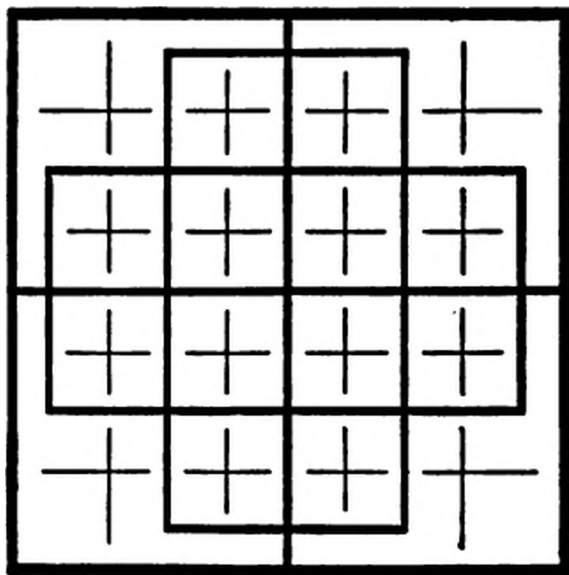
For *four* letters (which I call *a, b, c, d*) I use this Diagram; assigning the North Half to *a* (and of course the *rest* of the Diagram to *a'*), the West Half to *b*, the Horizontal Oblong to *c*, and the Upright Oblong to *d*.



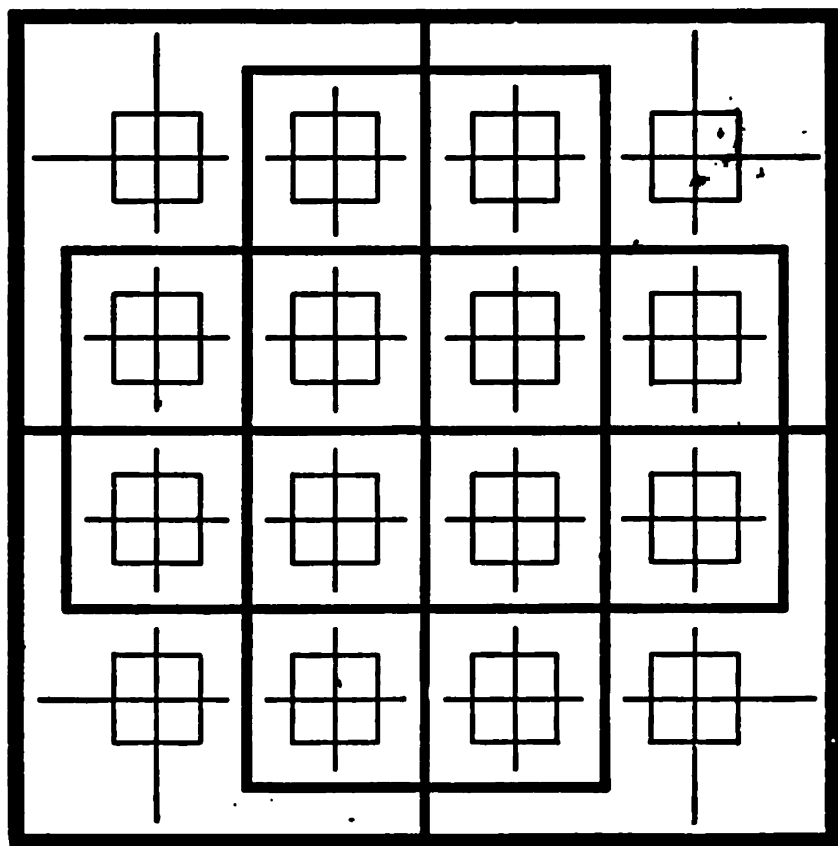
For *five* letters (adding *e*) I subdivide the 16 Cells of the previous Diagram by *oblique* partitions, assigning all the *upper* portions to *e*, and all the *lower* portions to *e'*. Here, I admit, we lose the advantage of having the *e*-Class all *together*, "in a ring-fence", like the other 4 Classes. Still, it is very easy to find; and the operation, of erasing it, is nearly as easy as that of erasing any other Class.



For *six* letters (adding *h*, as I avoid *tailed* letters) I substitute upright crosses for the oblique partitions, assigning the 4 portions, into which each Cell is thus divided, to the four Classes *eh, eh', e'h, e'h'*. We have now got 64 Cells.



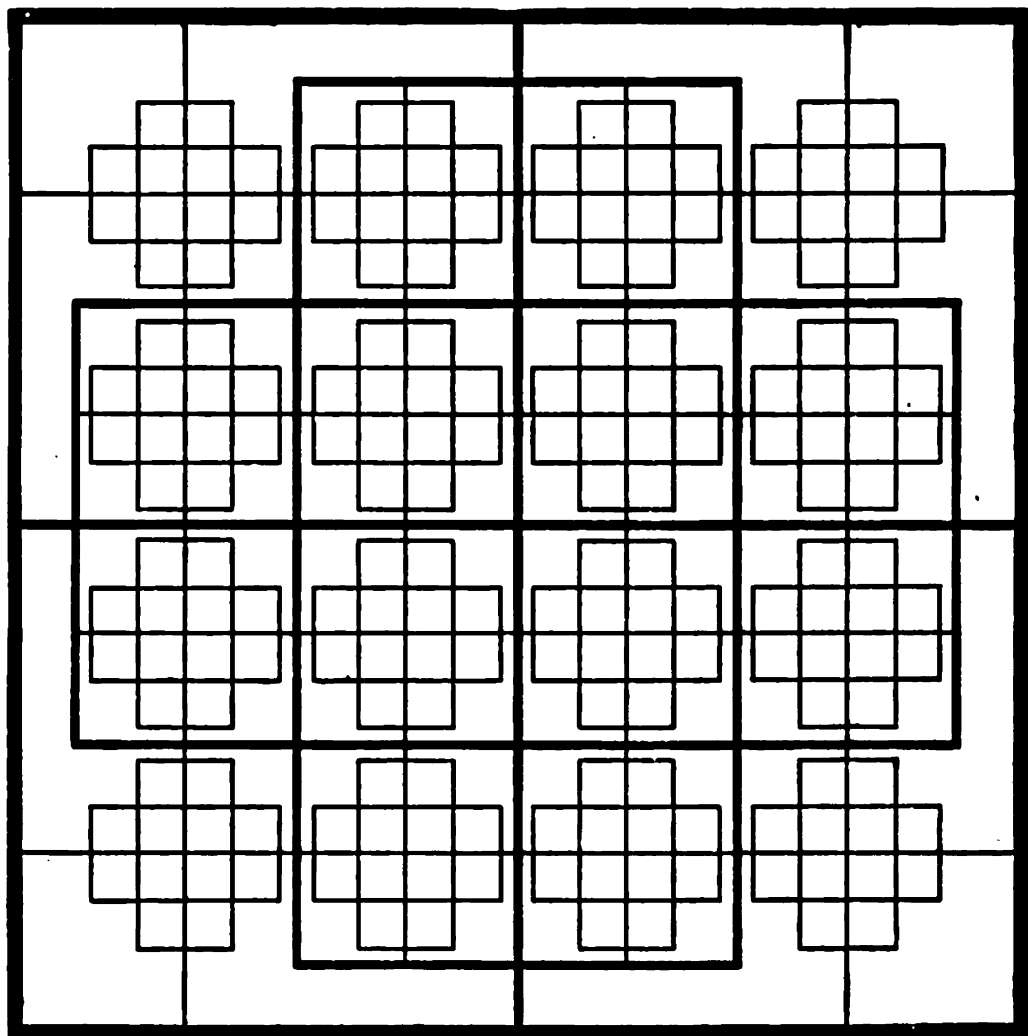
For *seven* letters (adding *k*) I add, to each upright cross, a little inner square. All these 16 little squares are assigned to



the *k*-Class, and all outside them to the *k'*-Class ; so that the 8 little Cells (into which each, of the 16 large Cells of the third Diagram, is divided) are respectively assigned to the 8 Classes *ehk*, *ehk'*, &c. We have now got 128 Cells.

For *eight* letters (adding *l*) I place, in each of the 16 large Cells of the third Diagram, a *lattice*, which is a miniature-copy of the whole Diagram ; and, just as the large Cells of the whole Diagram are assigned to the 16 Classes *abcd*, *abcd'*, &c., so the little Cells of each lattice are assigned to the 16 Classes *ehkl*, *ehkl'*, &c. Thus, the lattice in the N.W. corner serves to accommodate the 16 Classes *abc'd'ehk'l'*, *abc'd'eh'k'l'*, &c. This Octoliteral Diagram (see next page) contains 256 Cells.

For *nine* letters, I place 2 Octoliteral Diagrams side by side, assigning one of them to *m*, and the other to *m'*. We have now got 512 Cells.



Finally, for *ten* letters, I arrange 4 such Diagrams in a square, assigning them to the 4 Classes mn , mn' , $m'n$, $m'n'$. We have now got 1024 Cells.

§ 8.

Solution of a Syllogism by various Methods.

The best way, I think, to exhibit the differences between these various Methods of solving Syllogisms, will be to take a concrete example, and solve it by each Method in turn. Let us take, as our example, No. 29 (see p. 102).

“No philosophers are conceited ;

Some conceited persons are not gamblers.

∴ Some persons, who are not gamblers, are not philosophers.”

(1) *Solution by ordinary Method.*

These Premisses, as they stand, will give no Conclusion, as they are both negative.

If by 'Permutation' or 'Obversion', we write the Minor Premiss thus,

'Some conceited persons are not-gamblers,'

we can get a Conclusion in *Fresison*, viz.

"No philosophers are conceited ;

Some conceited persons are not-gamblers.

∴ Some not-gamblers are not philosophers"

This can be proved by reduction to *Ferio*, thus:—

"No conceited persons are philosophers ;

Some not-gamblers are conceited.

∴ Some not-gamblers are not philosophers".

The validity of *Ferio* follows directly from the Axiom '*De Omni et Nullo*'.

(2) *Symbolic Representation.*

Before proceeding to discuss other Methods of Solution, it is necessary to translate our Syllogism into an *abstract* form.

Let us take "persons" as our 'Universe of Discourse'; and let x = "philosophers", m = "conceited", and y = "gamblers."

Then the Syllogism may be written thus:—

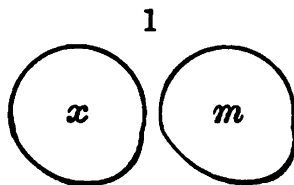
"No x are m ;

Some m are y '.

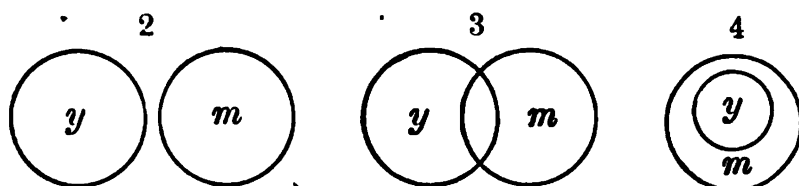
∴ Some y ' are x '."

(3) *Solution by Euler's Method of Diagrams.*

The Major Premiss requires only *one* Diagram, viz.

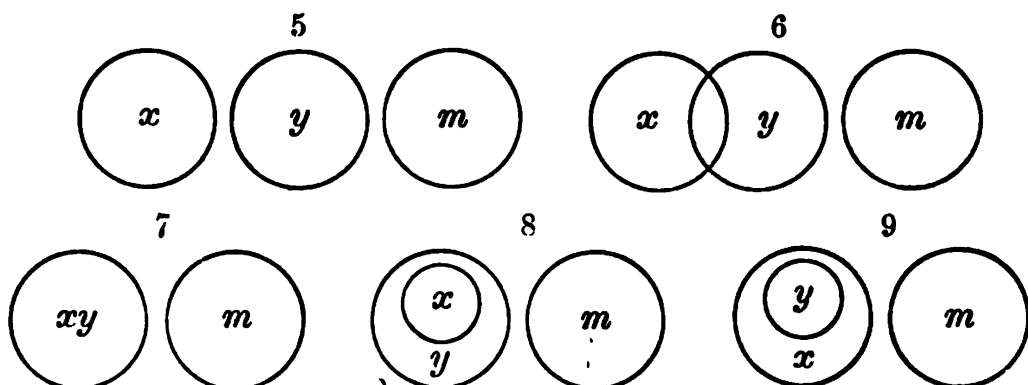


The Minor requires *three*, viz.

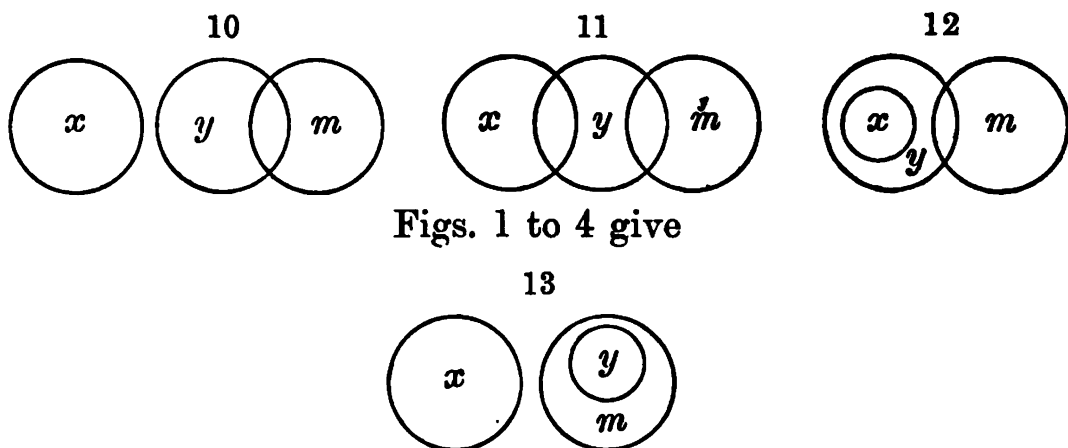


The combination of Major and Minor, in every possible way, requires *nine*, viz.

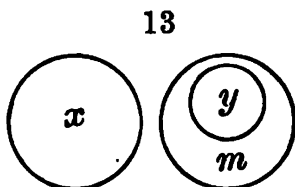
Figs. 1 and 2 give



Figs. 1 and 3 give

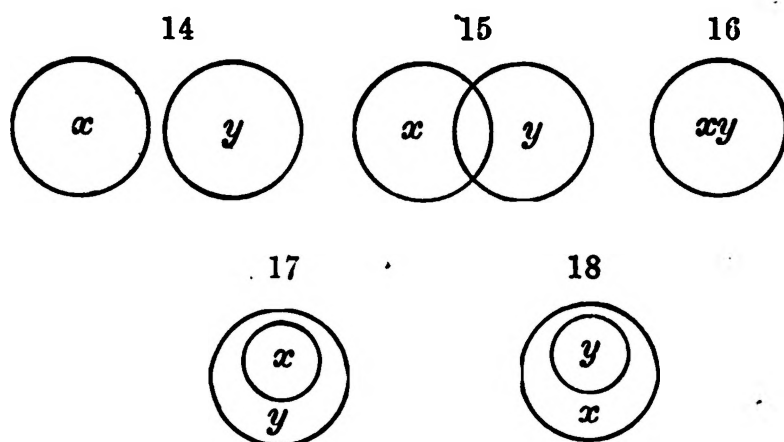


Figs. 1 to 4 give



From this group (Figs. 5 to 13) we have, by disregarding *m*, to find the relation of *x* and *y*. On examination we find that Figs. 5, 10, 13 express the relation of entire mutual exclusion; that Figs. 6, 11 express partial inclusion and partial exclusion; that Fig. 7 expresses coincidence; that Figs. 8, 12 express entire inclusion of *x* in *y*; and that Fig. 9 expresses entire inclusion of *y* in *x*.

We thus get five Biliteral Diagrams for x and y , viz.

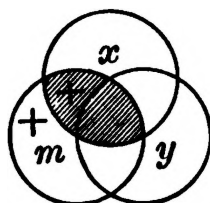


where the only Proposition, represented by them all, is "Some not- y are not- x ," i.e. "Some persons, who are not gamblers, are not philosophers"——a result which Euler would hardly have regarded as a *valuable* one, since he seems to have assumed that a Proposition of this form is *always* true!

(4) *Solution by Venn's Method of Diagrams.*

The following Solution has been kindly supplied to me by Mr. Venn himself.

"The Minor Premiss declares that some of the constituents in my' must be saved : mark these constituents with a cross.



The Major declares that all xm must be destroyed ; erase it.

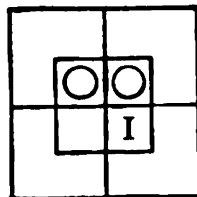
Then, as some my' is to be saved, it must clearly be $my'x'$. That is, there must exist $my'x'$; or, eliminating m , $y'x'$. In common phraseology,

'Some y' is x' ', or, 'Some not-gamblers are not-philosophers.'"

(5) *Solution by my Method of Diagrams.*

The first Premiss asserts that no xm exist : so we mark the xm -Compartment as empty, by placing a 'O' in each of its Cells.

The second asserts that some my' exist : so we mark the my' -Compartment as occupied, by placing a 'I' in its only available Cell.



The only information, that this gives us as to x and y , is that the $x'y'$ -Compartment is *occupied*, i.e. that some $x'y'$ exist.

Hence "Some x' are y' " : i.e. "Some persons, who are not philosophers, are not gamblers".

(6) *Solution by my Method of Subscripts.*

$$xm_0 \dagger my'_1 \P x'y'_1$$

i. e. "Some persons, who are not philosophers, are not gamblers."

§ 9.

My Method of treating Syllogisms and Sorites.

Of all the strange things, that are to be met with in the ordinary text-books of Formal Logic, perhaps the strangest is the violent contrast one finds to exist between their ways of dealing with these two subjects. While they have elaborately discussed no less than *nineteen* different forms of *Syllogisms*——each with its own special and exasperating Rules, while the whole constitute an almost useless machine, for practical purposes, many of the Conclusions being incomplete, and many quite legitimate forms being ignored——they have limited *Sorites* to *two* forms only, of childish simplicity; and these they have dignified with special *names*, apparently under the impression that no other possible forms existed !

As to *Syllogisms*, I find that their nineteen forms, with about a score of others which they have ignored, can all be arranged under *three* forms, each with a very simple Rule of its own ; and the only question the Reader has to settle, in working any one of the 101 Examples given at p. 101 of this book, is "Does it belong to Fig. I. II., or III.?"

As to *Sorites*, the only two forms recognised by the text-books, are the *Aristotelian*, whose Premises are a series of Propositions in *A*, so arranged that the Predicate of each is the Subject of the next, and the *Goclenian*, whose Premises are the very same series, written backwards. Goclenius, it seems, was the first who noticed the startling fact that it does not affect the force of a Syllogism to invert the order of its Premises, and who applied this discovery to a Sorites. If we assume (as surely we may?) that he is the *same* man as that transcendent genius who first noticed that 4 times 5 is the same thing as 5 times 4, we may apply to him what somebody (Edmund Yates, I think it was) has said of Tupper, viz., "here is a man who, beyond all others of his generation, has been favoured with Glimpses of the Obvious!"

These puerile—not to say infantine—forms of a Sorites I have, in this book, ignored from the very first, and have not only admitted freely Propositions in *E*, but have purposely stated the Premises in random order, leaving to the Reader the useful task of arranging them, for himself, in an order which can be worked as a series of regular Syllogisms. In doing this, he can begin with *any one* of them he likes.

I have just tabulated, for curiosity, the various orders in which the Aristotelian Sorites

1. All *a* are *b* ;
2. All *b* are *c* ;
3. All *c* are *d* ;
4. All *d* are *e* ;
5. All *e* are *h* ;

may be syllogistically arranged, and I find there are no less than *sixteen* such orders, viz., 12345, 21345, 23145, 23415, 23451, 32145, 32415, 32451, 34215, 34251, 34521, 43215, 43251, 43521, 45321, 54321. Of these the *first* and the *last* have been dignified with names; but the other *fourteen*—first enumerated by an obscure Writer on Logic, towards the end of the Nineteenth Century—remain without a name!

§ 10.

Some account of Parts II, III.

In Part II. will be found some of the matters mentioned in this Appendix, viz., the “Existential Import” of Propositions, the use of a *negative* Copula, and the theory that “two negative Premisses prove nothing.” I shall also extend the range of Syllogisms and of Sorites, by introducing Propositions containing *alternatives* (such as “Not-all x are y ”), Propositions containing 3 or more Terms (such as “All ab are c ,” which, taken along with “Some bc' are d ”, would prove “Some d are a' ”), and Sorites containing Premisses of other forms than Propositions in E and in A . I shall also discuss the *very* puzzling subjects of “Hypotheticals,” Dilemmas, and Paradoxes. I hope, in the course of Part II., to go over all the ground usually traversed in the text-books used in our Schools and Universities, and to enable my Readers to solve Problems of the same kind as, and far harder than, those that are at present set in their Examinations.

In Part III. I hope to deal with many curious and out-of-the-way subjects, some of which are not even alluded to in any of the treatises I have met with. In this Part will be found such matters as the Analysis of Propositions into their Elements (let the Reader, who has never gone into this branch of the subject, try to make out for himself what *additional* Proposition would be needed to convert “Some a are b ” into “Some a are bc ”), the treatment of Numerical and Geometrical Problems, the construction of Problems, and the solution of Syllogisms and Sorites containing Propositions more complex than any that I have used in Part II.

I will conclude with a couple of Sorites, as a taste of what is coming. I shall be very glad to receive, from any Reader, who thinks he has solved either of them, what he conceives to be its complete Conclusion.

It may be well to explain what I mean by the complete Conclusion of a Syllogism or a Sorites. I distinguish their Terms as being of two kinds—those which can be eliminated

(e.g. the Middle Term of a Syllogism), which I call the "Eliminands," and those which *cannot*, which I call the "Retinends"; and I do not call the Conclusion *complete*, unless it states *all* the relations, among the Retinends only, which can be deduced from the Premisses.

1.

- (1) A logician, who eats pork-chops for supper, will probably lose money ;
- (2) A young man always gets up at 5 a.m., unless he has lost money ;
- (3) No earnest man, who ^{who does not gamble and} does not eat pork-chops for supper, need take to cab-driving, ~~unless he gambles.~~ ;
- (4) A logician, who is in danger of losing money, had better take to cab-driving ;
- (5) A gambler, whose appetite is not ravenous, will probably lose money ;
- (6) A man who is depressed, having lost money and being likely to lose more, always rises at 5 a.m. ;
- (7) A man, who neither gambles nor eats pork-chops for supper, is sure to have a ravenous appetite ;
- (8) A lively man, who goes to bed before 4 a.m., had better take to cab-driving ;
- (9) A man with a ravenous appetite, who has not lost money and does not rise at 5 a.m., always eats pork-chops for supper ;
- (10) An earnest gambler, who is depressed though he has not lost money, is in no danger of losing any ;
- (11) A man, who does not gamble, and whose appetite is not ravenous, is always lively ;
- (12) A lively logician, who is really in earnest, is in no danger of losing money ;
- (13) A man with a ravenous appetite has no need to take to cab-driving, if he is really in earnest ;
- (14) A gambler, who is depressed though in no danger of losing money, sits up till 4 a.m.
- (15) A man, who has lost money and does not eat pork-chops for supper, had better take to cab-driving, unless he gets up at 5 a.m.

[TURN OVER

- (16) A gambler, who goes to bed before 4 a.m., need not take to cab-driving, unless he has a ravenous appetite ;
- (17) An old man, who does not gamble, and who has not lost money though he is in danger of doing so, always eats pork-chops for supper ;
- (18) A man with a ravenous appetite, who is depressed though in no danger of losing money, is a gambler.
- Univ. "men" ; *a* = earnest ; *b* = eating pork &c. ; *c* = gamblers ; *d* = getting up at 5 ; *e* = had better take &c. ; *h* = has lost &c. ; *k* = has ravenous &c. ; *l* = lively ; *m* = logicians ; *n* = sitting up till 4 ; *r* = will probably lose &c. ; *s* = young.

2.

There are 5 friends, *A*, *B*, *C*, *D*, and *E*, each of whom makes 2 assertions, so that it is possible for him to tell 2 Truths, or a Truth and a Lie, or 2 Lies.

A says " Either *B* or *D* tells a Truth and a Lie : either *C* or *E* tells 2 Lies."

B says " Either *A* or *C* tells a Truth and a Lie : either *D* tells 2 Lies or *E* tells 2 Truths."

C says " Either *A* or *D* tells 2 Truths : either *B* tells a Truth and a Lie or *E* tells 2 Lies."

D says " Either *A* or *E* tells 2 Truths : either *B* tells 2 Lies or *C* tells 2 Truths."

E says " Either *A* or *B* tells 2 Lies : either *C* or *D* tells a Truth and a Lie."

What is the condition, as to truth-telling and lying, of each of these five painfully-candid friends ?

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